UNIVERSIDAD COMPLUTENSE DE MADRID

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# TIME MICROECONOMICS ON THE ALLOCATION OF TIME AND CHOICE OVERLOAD|

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# Time Microeconomics

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# Chapter 1

# Introduction

> *To be without some of the things you want is an indispensable part of happiness.*
>
> (Bertrand Rusell)

## 1.1 Purpose and goals

*Economics is the science which studies human behaviour as a relationship between ends and scarce means which have alternative uses* (Robbins, 1932). Everyone would agree that the main resource, in the ultimate application of the definition of economics, is time. The scarce time that is given to us is probably the resource with more alternative uses in order to pursue our ends.

It is however remarkable how the role of time and its use have been underestimated in (micro) economics. This dissertation focuses on the use of time behind some decision problems and intends to highlight its relevance. In particular we address some problems considered in the economic literature concerning the allocation of time. Moreover, we use time as the means to explain some other interesting phenomena introduced in other fields (e.g. social psychology), related with individual's welfare when facing a choice problem with many alternatives. Somehow unexpectedly it has been suggested that more choice does not necessarily imply more welfare. Here we provide an explanation for some choice overload phenomena in terms of the use of time.

Concretely, the main goal of this dissertation is twofold: first, to formulate an extended theory of allocation of time which takes into account the problem of joint production noted in economic literature; second, to provide a formal theoretical explanation about some choice overload problems. A third goal consists of exhibiting empirical applications of our theoretical approach to explain choice overload in a practical setting.

## 1.2 Some literature

### 1.2.1 Concerning microeconomic theory and time use

Economic theory did poorly cover the topic called allocation of time before the 1960s. Nevertheless, we find in Reid (1934) and Mincer (1962) the first mentions to a time use arguments linked to economics. However, no model was apparently able to include ideas revolving around the use of time and microeconomic theories.

In parallel, some other economists had been thinking of a new way to approach microeconomic behaviour, in contrast to the standard economic model[1]. So a theoretical modelling can be found in the doctoral dissertation by Duncan Ironmonger, defended at the University of Cambridge in 1962, however published ten years later, in 1972. A similar theoretical setting is proposed by Lancaster (1966). Nonetheless, neither Ironmonger (1972) nor Lancaster (1966) refer explicitly to time as a central input in their models[2]. Their models deal with the maximization of utility, defined over commodities or wants, which are obtained by using different inputs or characteristics. Basically, Ironmonger (1972) argues that in order to produce each want some inputs are required, while Lancaster (1966) states that each good is different if the characteristics are different.

The particularization of the setting in Ironmonger (1972) to the issue of allocation of time came with Becker (1965) in his well-known *theory of allocation of time*. Becker (1965) argues that consumers maximize utility, which is defined over what calls commodities that are produced with market goods and time, and consumer faces both budget and time constraints.

[1]Based on maximizing utility defined over market goods, given that consumer has limited resources and market goods are costly at market prices in a competitive market.

[2]Reid (1934) is considered the genesis of this line of thought suggesting a new way to approach microeconomic behaviour and is also considered an antecedent of Becker (1965). However the contribution by Margaret Reid has not been very highlighted by a quite considerable part of related economic literature.

Becker (1965) has generated large research in the social sciences; in economics and sociology, particularly. However, theoretical contributions in economic theory have not been so abundant since his trigger paper. Some exceptions are the papers by DeSerpa (1971) or Evans (1972), that can be seen in essence as particular cases of Becker (1965).

The contribution by Pollak and Wachter (1975) can be considered the first critique to Becker's model. They posed some problems of time use models in general, and in particular of the benchmark model proposed by Becker (1965). Pollak and Wachter (1975) left some open questions that will be addressed in chapter 2 in this dissertation. Pollak and Wachter (1975) generated some interesting direct replies, like Barnett (1977).

Another perspective is given by Gronau (1977), which uses a very simple theoretical model to provide interesting insights and interpretations of real situations supported by empirical information. Some other discussions are Flemming (1973) and Juster and Stafford (1991). Particularly interesting is the survey paper by Juster and Stafford (1991), an up-to-date account of both theoretical and empirical research on the matter.

Little research on time use has employed dynamic models. Fischer (2001) analyses procrastination using time inputs as a key variable. Gonzalez-Chapela (2004) comprises essays on time allocation dealing with dynamic models. A particular model can be found in Gonzalez-Chapela (2007).

In chapter 2 we suggest an extended theory of allocation of time, after Becker (1965). Such extension takes into account the problem of joint production, noted in Pollak and Wachter (1975). Moreover, we illustrate how this theory may work through an exercise on the retirement age policies.

### 1.2.2 Concerning the choice overload and the paradox of choice

In the last decade an increasing number of researchers have paid attention to what is known as choice overload problems and, in particular, the so called paradox of choice. In essence, choice overload and the paradox of choice deal with the following idea: increasing the number of options within a choice set may create more problems than benefits for the choice maker.

The trigger publications can be considered Iyengar and Lepper (2000) and Schwartz (2000). Both, simultaneously and we would say, complementarily, provide us with the following contributions: a nice dissertation on the paradox of choice from the psychological perspective (Schwartz, 2000) and some empirical information in favour of the existence of the choice overload throughout field studies and laboratory experiments (Iyengar and Lepper, 2000).

The topic has called the attention both in psychology and in economics, in particular in the field of marketing and business economics. During the last five years the topic has grown through a number of papers, as with Schwartz (2004, 2005, 2006) or Mogilner et al. (2008). Choice overload has called the attention of experimental and behavioural economics, as Reutskaja (2008), Reutskaja and Hogarth (2009), Reutskaja et al. (2011) or Caplin et al. (2012). More recently, the main issue has been enriched through some controversy in empirical evidence supporting choice overload (Chabris et al., 2009; Scheibehenne et al., 2010), theoretical discussions combined with empirical arguments (Dar-Nimrod et al., 2009; Markus and Schwartz, 2010; Grant and Schwartz, 2011), and applications to hot topics in economics (Iyengar and Kamenica, 2010; Kamenica et al., 2011).

In chapter 3 a simple model of time use –that can be seen as a particular case of the theory in chapter 2– is introduced to deal with choice overload situations described by social psychologists. Time variables have been proposed as relevant whenever choice overload is discussed. We obtain formal results, under a time use perspective, about several choice overload phenomena (mainly the paralysis effect and the paradox of choice); our results are obtained using an economic analysis approach.

Subsequently, in chapter 4 we pursue our third goal described above: to show empirical applications of the results obtained in chapter 3. We carry out a numerical analysis of the model in chapter 3, and also we apply this numerical analysis to a case study with actual data prices for different available options. Our numerical results seem robust, and suggest evidence in favour of our theoretical results concerning choice overload.

# Chapter 2

# Review of the theory and a possible extension[1]

*He who loves practice without theory is like the sailor who boards ship without a rudder and compass and never knows where he may cast.*

(Leonardo Da Vinci)

This chapter reviews different economic models which include time use in an explicit and endogenous manner. In section 2.1 the main features of existing time use models in economics are presented. Then, in section 2.2 we introduce some significant problems that this kind of well-established models entail; the most highlighted being the so called joint production problem. Subsequently, in section 2.3 we propose an extended theory which takes into account the issue of joint production. Last, we illustrate how our theory can be applied through a problem related with retirement.

[1] An earlier version of this chapter has been published in Sanchis (2013).

## 2.1 Mainstream time use models in microeconomics

In this section we illustrate how classic microeconomic models incorporate time use as a choice variable to provide a more realistic perspective of some decision problems. Firstly, we present a simple possible version of the leisure model. Secondly, we show a simple version of the Becker model.

Economic theory has produced a well known textbook model, frequently used in labour economics among other areas. Such model, often known as leisure model, includes the use of time as a choice variable and can be presented as follows, in line with the notation in Varian (1992) or Mas-Colell et al. (1995):

$$\begin{cases} \max\limits_{x,T_1} & U = U(x, T_1), \\ s.t. & px \leq w(T - T_1) + V, \end{cases} \tag{2.1}$$

where $T_1$ is the leisure time, $w$ is the wage per unit of time and $V$ is the non-labour income, $x$ is consumption and $p$ is the price of such consumption.

The solution of this model is obtained by the usual marginalist analysis, and in essence deals with leisure time as an extra good that in practice is as if it were purchased in the market at the wage rate.

A simplified version of the benchmark model by Becker (1965) is as follows. The main innovation is the introduction of what Becker calls *commodities*, which determine utility; such commodities are either tangible (home-made products) or intangible (home-made services, personal needs or similar) outputs produced with inputs such as time use and market products. Consider two commodities, $Z_1$ and $Z_2$, where $Z_1$ is produced with time $\vec{T_1}$ and goods $\vec{x_1}$, and $Z_2$ is also produced with time $\vec{T_2}$ and goods $\vec{x_2}$. An individual has to work at a wage $w$ the remaining time, and may have non-labour income $V$ at her disposal. She solves the problem below to determine $\vec{x_1}, \vec{T_1}, \vec{x_2}, \vec{T_2}$, i.e. the quantities of market goods[2] and the time allocations[3].

[2]Each component of each vector of goods is denoted as $x_{ij}$, which is the $j$-th market good used in the production of the $i$-th want or commodity, for all $i, j$.

[3]Similarly, each component of each vector of time inputs is denoted with $T_{ij}$, which is the amount of the $i$-th type or aspect of time spent in the production of $j$-th want or commodity, for all $i, j$. The components of $\vec{T}$ are denoted with $T_i$, for all $i$, which represent the total amount of time existing for each type or aspect of time, which is exogenously given, and the sum of all $T_i$ for all $i$ is $T$, the total amount of time which has to be allocated in the problem.

$$\left\{ \begin{array}{rl} \max\limits_{\vec{x_1},\vec{x_2},\vec{T_1},\vec{T_2}} & U = U(Z_1(\vec{x_1},\vec{T_1}), Z_2(\vec{x_2},\vec{T_2})), \\ s.t. & \vec{p_1}^T\vec{x_1} + \vec{p_2}^T\vec{x_2} \leq \vec{w}^T(\vec{T} - \vec{T_1} - \vec{T_2}) + V, \\ & \vec{T_1} + \vec{T_2} \leq \vec{T}, \\ & \vec{T_i} = (T_{ij})_j \ with \ T_{ij} \geq 0, \text{for all } i, j. \end{array} \right. \tag{2.2}$$

As a simple example, if we set a meal as $Z_1$ and listening to music as $Z_2$, this model would require as $\vec{x_1}$ the vector of ingredients of the meal, let us say, meat ($x_{11}$) and potatoes ($x_{21}$). In addition, in order to produce and enjoy a meal, time is needed: the vector of time inputs $\vec{T_1}$ may be composed by the cooking time in the kitchen ($T_{11}$) and the eating time in the dining room ($T_{21}$). Similarly, in order to listen to music ($Z_2$), a vector of goods ($\vec{x_2}$) composed by a CD player ($x_{12}$) and an album in CD format ($x_{22}$) is required, but also a vector of time ($\vec{T_2}$) composed by listening to music in the kitchen ($T_{12}$) and listening to music in the dining room ($T_{22}$). We thus have

$$\vec{x_1}_{(2\times1)} = \begin{pmatrix} x_{11} \\ x_{21} \end{pmatrix},$$

$$\vec{x_2}_{(2\times1)} = \begin{pmatrix} x_{12} \\ x_{22} \end{pmatrix},$$

$$\vec{T_1}_{(2\times1)} = \begin{pmatrix} T_{11} \\ T_{21} \end{pmatrix},$$

$$\vec{T_2}_{(2\times1)} = \begin{pmatrix} T_{12} \\ T_{22} \end{pmatrix},$$

$$\vec{T}_{(2\times1)} = \begin{pmatrix} T_1 \\ T_2 \end{pmatrix},$$

with $T_1 + T_2 = T$.

Of course all the inputs of time must add up to the total time available $T$ and the goods employed must be feasible. This analysis aim at a more detailed description and explanation of the consumer behaviour.

Notice that the model *à la* Becker is more general than the leisure model: first, the concept of commodities expands the arguments in the utility function and, second, the model *à la* Becker defines different types of time (or as Becker termed them, aspects). The model in (2.2) collapses to (2.1) if $Z_1 = x$ –whose price is $p$– and $Z_2 = T_1$, where there obviously is just one type or aspect of time.

## 2.2 Major theoretical problems of time use models

The improvements in realism expected from time use models bring several challenges which still remain as theoretical obstacles. This section comments on the main problems, according to the literature in economics. We start by summarizing achievements of the benchmark model by Becker (1965), to focus on its critique later on.

The Becker's model is the following:

$$\begin{cases} \max\limits_{\vec{x_i},\vec{T_i}} & U = U(Z_1(\vec{x_1},\vec{T_1}),...,Z_m(\vec{x_m},\vec{T_m})), \\ s.t. & \sum\limits_{i=1}^{m} \vec{p_i}^T \vec{x_i} \leq \vec{w}^T \vec{T_w} + V, \\ & \sum\limits_{i=1}^{m} \vec{T_i} = \vec{T} - \vec{T_w}, \\ & \vec{T_i} = (T_{ij})_j \ with \ T_{ij} \geq 0, \text{ for all } i,j. \end{cases} \tag{2.3}$$

Notice that this model includes the vector of commodities or wants $\vec{Z}$ as an argument in the utility function ($U$). Each want ($Z_i, i = 1,...,m$) is obtained by using some *ingredients*, let us say, such as a vector of goods $\vec{x_i}$ and a vector of time inputs $\vec{T_i}$.

Becker (1965) merges the budget constraint and the time constraints into what he named as the full income constraint:

$$\sum_{i=1}^{m} \vec{p_i}^T \vec{x_i} + \sum_{i=1}^{m} \vec{w}^T \vec{T_i} \leq \vec{w}^T \vec{T} + V.$$

At this point Becker (1965) makes some strong assumptions:

$$\vec{x}_{i(n\times 1)} = \vec{b}_{i(n\times 1)} Z_i,$$

$$\vec{T}_{i(p\times 1)} = \vec{t}_{i(p\times 1)} Z_i,$$

where $\vec{b_i}$ and $\vec{t_i}$ are the vectors giving the input of goods and time, respectively, per unit of $Z_i$. Notice that these assumptions impose linear relations defined by fixed coefficients.

Therefore, the model can be rewritten in this alternative way:

$$\begin{cases} \max\limits_{\vec{Z}} & U = U(Z_1, ..., Z_m), \\ s.t. & \sum\limits_{i=1}^{m} \pi_i Z_i \leq \vec{w}^T \vec{T} + V = S, \end{cases} \tag{2.4}$$

where $\pi_i$ would represent –following the Becker (1965) terminology– the *full price* of each unit of commodity $Z_i$. Such *full price* would include the value of both goods and time used for such commodity, as follows:

$$\pi_i = \vec{p_i}^T \vec{b_i} + \vec{w}^T \vec{t_i}.$$

This nice alternative version of the model in (2.3) permits to solve the problem for $Z_i$'s, and in turn to do comparative statics as in the classical microeconomic textbook model.

Pollak and Wachter (1975) observed two shortcomings in the model by Becker (1965): absence of joint production and the need of constant returns to scale in the production of each $Z_i$.

It is apparent that the model cannot apply to many simple situations. Let us think in terms of the example of cooking and listening to music from the previous section: the time devoted to listen to music in the kitchen cannot be the same (simultaneous) as the time spent cooking, which is something very realistic for many cooks who cook while they listen to music. This feature is known in the literature as *joint production*, and it was first noticed by Pollak and Wachter (1975).

The need of constant returns to scale in the production of each $Z_i$ is related to the way in which the model is converted from (2.3) to (2.4). As we already mentioned, linear relations are assumed in the production of each commodity in relation to both goods and time. The model is thus transformed into one in which each commodity or want has a price ($\pi_i$); therefore, the consumer must choose her desired level of each want or commodity taking into account that prices for such wants and commodities are well defined by the $\pi_i$'s. The budget constraint is substituted into what is called the *full income constraint*. The problem of constant returns to scale in the production of wants is a technical discussion established by Pollak and Wachter (1975). This discussion led them to conclude the following: in order to get a simplified model in which each commodity has a price ($\pi_i$) which is independent from the choice variable of the problem ($Z_i$), the production of each commodity must satisfy constant returns to scale, and joint production is not possible. Otherwise, the model cannot be rewritten as in (2.4).

The problem of joint production is even more tricky, apart from its influence in the issue described above. Pollak and Wachter (1975) discuss extensively the issue of joint production, –since it creates a more structural problem than analytical–, and provide bright insights; however no theoretical model solving joint production is given. The issue remains unsolved in the literature; it also creates numerous problems when researchers work with time use data, because of –as they call it– simultaneous activities, i.e. joint production. The following quotation illustrates this importance:

> *"The major problem in studying the allocation of time in the household production function model is centred on joint production rather than non-constant returns to scale"* (Pollak and Wachter, 1975).

There are other problems as the one suggested by DeSerpa (1971), which argues that consumption of goods are constrained by some minimum amount of time that is needed for such consumption. Therefore, extra constraints must be added to the Becker model. Although DeSerpa (1971) is in essence a particular case of Becker (1965), it poses a plausible problem and also proposes its solution.

## 2.3 Extended model of allocation of time

In order to facilitate the presentation of the extended model, we will refer to the cooking example above when commenting on joint production issues.

Let us define the production of wants, –which are represented by the $m$-dimensional vector $\vec{Z} = (Z_1, ..., Z_m) \in \Re^m$– as follows, for all $i = 1, ..., m$:

$$Z_i = f_i \left( X_{n \times q}, \Im_{p \times r} \right), \tag{2.5}$$

where

$$X_{n \times q} = \begin{pmatrix} x_{11} & \cdots & x_{1q} \\ \vdots & & \vdots \\ x_{n1} & \cdots & x_{nq} \end{pmatrix},$$

$$\Im_{p \times r} = \begin{pmatrix} T_{11} & \cdots & T_{1r} \\ \vdots & & \vdots \\ T_{p1} & \cdots & T_{pr} \end{pmatrix}.$$

The cooking example would imply a setting in which $Z_1$ and $Z_2$ would be functions of the following form,

$$Z_i = f_i \left( X_{2 \times 2}, \Im_{2 \times 1} \right), i = 1, 2. \tag{2.6}$$

This simply states that each want may be produced as a function of all the ingredients and musical components (the goods), however both commodities can be produced using the same time inputs, which are the cooking time $T_{11}$ and the eating time $T_{21}$. It can be specified that for this particular example, the meal can be produced defining the matrix $X$ only over the vector of ingredients, whereas the listening to music can be produced using just the vector of music components, following the Becker specification. However, Becker specification cannot model this situation in which both when cooking and when enjoying the meal the consumer is listening to music, which is what the setting in (2.5), and of course in (2.6), allows for.

Therefore, we can propose the extended model of allocation of time[4], by implementing the setting in (2.5) into a Becker-based model, as follows:

$$\begin{cases} \max\limits_{X,\Im} & U = U\left(Z_1(X_{n\times q}, \Im_{p\times r}), ..., Z_m(X_{n\times q}, \Im_{p\times r})\right), \\ s.t. & G\left(X_{n\times q}, \Im_{p\times r}\right) \leq \vec{w}^T \vec{T_w} + V, \\ & \sum\limits_{k=1}^{r} \vec{T_k} = \vec{T}, \\ & \sum\limits_{l=1}^{p} T_l = T, \\ & X_{n\times q} \geq 0_{n\times q}, \Im_{p\times r} \geq 0_{p\times r}, \end{cases} \tag{2.7}$$

where

- $G$ is a function which express the expenditure of resources made by this individual, in terms of money.
- $\vec{x_j}$ is a vector corresponding to the $j$-th column in $X_{n\times q}$. where $j$ is a generic use of the goods.
- $\vec{T_k}$ is a vector corresponding to $k$-th column in $\Im_{p\times r}$, where $k$ is a generic use of time.
- $\vec{T}$ is a $p$-dimensional vector whose elements, $T_l$, represent the amounts of time available for each type of time $l$.

[4]A very preliminary version of this model was presented in the 29th IATUR Conference. 17-19 Oct 2007, Washington D.C., USA., which was included in Sanchis (2007).

- $T$ is the total immutable time available (24 hours per day, 7 days a week, etc)
- $\vec{w}$ is the $p$-dimensional vector of wage rates for any type of time.
- $\vec{T_w}$ is the $p$-dimensional vector of working time for any type of time. Note that this specific use of time is included in the matrix $\Im_{p\times r}$ within all the $r$ uses of time.
- $V$ is any other income which does not comes from $\vec{T_w}$.

DeSerpa (1971) introduces a set of linear constraints as fixed proportions of minimum time needed for the consumption of each market goods. These constraints take a generic form as $T_{pr} \geq \alpha_h x_{nq}$. To take into account the minimum time assumption posed in DeSerpa (1971), notice that any extra linear constraint can be modelled with this set of constraints:

$$B_{s\times(nq+pr)}Q_{(nq+pr)\times 1} \leq 0_{s\times 1},$$

where

- $B_{s\times(nq+pr)}$ is a matrix of positive or negative coefficients (all elements equal to zero implies the presence of no extra constraint), and
- $Q_{(nq+pr)\times 1} = (x_{11}, \ldots, x_{nq}, T_{11}, \ldots, T_{pr})$

It must be noticed that the matrix of time inputs can be considered as a grid in which each type of time is, say, a five-minutes slot, showed vertically, as a schedule. Each row in the matrix would be a 5-minute slot within the 24 hours of the day. All possible ways in which time can be used could be arranged by columns. This setting would account for joint production, e.g. I can be driving and listening to music between 8:11h and 8:15h, as well as cooking and listening to music between 20:11h and 20:15h. A parallel interpretation can be done for the goods.

A common way to express the left hand side of the budget constraint in (2.7) is as the expenditure in market goods. The model thus reads as follows:

$$
\begin{cases}
\max\limits_{X,\Im} & U = U(Z_1(X_{n\times q}, \Im_{p\times r}), ..., Z_m(X_{n\times q}, \Im_{p\times r}), \\
s.t. & \sum\limits_{j=1}^{q} \vec{p}^T \vec{x_j} \leq \vec{w}^T \vec{T_w} + V, \\
& \sum\limits_{k=1}^{r} \vec{T_k} = \vec{T} - \vec{T_w}, \\
& \sum\limits_{l=1}^{p} \vec{T_l} = \vec{T}, \\
& X_{n\times q} \geq 0_{n\times q}, \Im_{p\times r} \geq 0_{p\times r}.
\end{cases}
\tag{2.8}
$$

Of course this way to express market expenditure could be replaced by any other expenditure function $G$.

It is obvious that Becker model is a particular case of the last problem (2.8), when the production of the $m$-th commodity or want is only depending on the $m$-th column of both $X_{n\times q}$ and $\Im_{p\times r}$, $m = q = r$ and no extra constraints are regarded.

## 2.4 A simple illustration

We present a simple application which illustrates the potential use of the extended model suggested earlier. We do it by applying the model to a very simple life cycle model based on time use; our purpose is to analyse the impact of policies in favour of increasing the retirement age. This life-cycle perspective we adopt in this application makes unavoidable the presence of joint production, and so we illustrate how our extended model behaves since Becker (1965) cannot be used for this purpose.

### 2.4.1 Increase in the retirement age?

This section[5] illustrates one possible application of a simple version of the extended model. This application is related to the research in line with Heckman (1976), although with a static model. The core idea of the model

[5]This application was presented at the 32nd IATUR Conference: Time-Budgets and Beyond: The Timing of Daily Life. 7-10 Jul 2010, Paris, France.

can be said to be partially described with the empirical job in Easterlin (2006) and Bonke et al. (2009).

Our simple application seeks to model the following idea, presented as an apparently unrealistic or very abstract example. A policy maker tries to influence individual's decision over the use of time. Nevertheless, the policy maker just can control in some degree some parameters for the individual (as the total amount of time available, wages, and others). Of course, the individual decides on her use of time and has her own tastes and preferences; all these choices comprise what is known as the life-cycle. This sort of debate about policies in favour of increasing the retirement age is very controversial (and its currently taking place in e.g. France and Spain).

The implications for policy makers can be found in issues like retirement decisions under lifetime choice, retirement decision "in the margin" (anticipated retirement), work-life balance over the working (lifetime) period, or daily work-life balance in terms of time budgets.

The model is applied here to a work-life balance over the working lifetime period, e.g. 16-65 year old period, in which an individual must decide how much time to work in the labour market; the policy maker is considering to increase the retirement age up to 67 years, in order to get higher working time in the labour market by the individuals for, say, fiscal reasons.

### The Model

An individual's lifetime decisions can be fundamentally related to how much time this individual is willing to allocate into some activity throughout a considerable period of her life. With this in mind, this individual must solve a very simple, abstract and maybe unrealistic problem like

$$\left\{ \begin{array}{rl} \max\limits_{T_{11},T_{12}} & U(Z_1(T_{11},T_{12}),Z_2(T_{11},T_{12})), \\ s.t. & G \leq w_{11}T_{11} + w_{12}T_{12} + V, \\ & T_{11} + T_{12} = T, \end{array} \right. \tag{2.9}$$

where there is only one type of time, the work life ($T_1 \equiv T$), -determined by the government-, which can be spent in working ($T_{11}$) and not working

($T_{12}$). The partition (work-life balance) of work life in both uses produces *jointly* job satisfaction ($Z_1$) and personal satisfaction ($Z_2$). Individual's expenditure for the lifetime period of working time is denoted by $G$; so far, we will consider $G$ as a fixed amount of money for the whole lifetime period of working life, although later on we will relax this assumption. Average wage rate per unit of working time is denoted with $w_{11}$, while average subsidized income obtained from the welfare state during the non-working time periods is denoted by $w_{12}$. All other non-labour income is represented by $V$.

The model in (2.9) can be reduced to a model in which the decision variable is the working time within the total time available, as follows:

$$\begin{cases} \max\limits_{T_{11}} & U(Z_1(T_{11}, T - T_{11}), Z_2(T_{11}, T - T_{11})), \\ s.t. & G \leq (w_{11} - w_{12})T_{11} + w_{12}T + V, \\ & 0 \leq T_{11} \leq T. \end{cases} \tag{2.10}$$

Assume that $w_{11}T + V \geq G$, so that working the full work life guarantees the minimum expenditure level $G$. In the typical case that $w_{11} > w_{12}$, this implies that

$$T_{11}^{min} \equiv \frac{G - V - w_{12}T}{w_{11} - w_{12}} \leq T_{11} \leq T.$$

This defines the feasible set for the retirement problem. There are two possible solutions, depending upon whether the budget constraint binds or not. When it does not bind, an interior solution must satisfy

$$\sum_{i=1}^{2} \frac{\partial U}{\partial Z_m} \frac{\partial Z_m}{\partial T_{11}} = 0,$$

with $\dfrac{G - V - w_{12}T}{w_{11} - w_{12}} \leq T_{11}^* \leq T.$

On the other hand, when the budget constraint binds – also assuming $w_{11} > w_{12}$– there are two possible solutions: $T_{11}^* = T$ or $T_{11}^* = T_{11}^{min}$.

Assume that

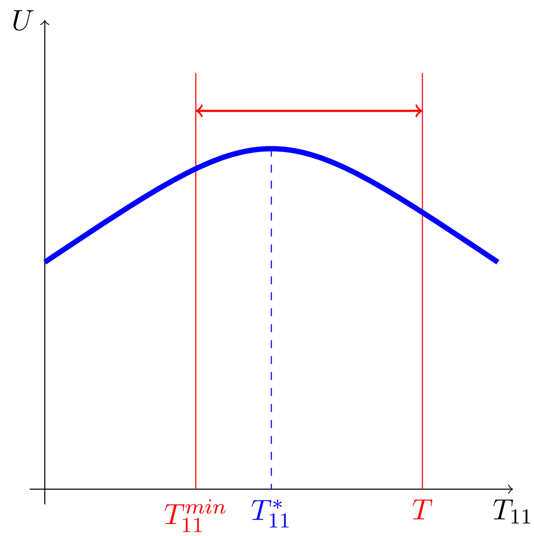


Figure 2.1: Solution when the budget constraint does not bind

$$\frac{\partial U}{\partial Z_i} > 0, i = 1, 2.$$

Notice that

$$\begin{aligned}\frac{\partial U}{\partial T_{11}} = & \frac{\partial U}{\partial Z_1}\left(\frac{\partial Z_1}{\partial T_{11}} - \frac{\partial Z_1}{\partial T_{12}}\right) + \frac{\partial U}{\partial Z_2}\left(\frac{\partial Z_2}{\partial T_{11}} - \frac{\partial Z_2}{\partial T_{12}}\right) = \\ & = JMU_{T_{11}} - JMU_{T_{12}}\big|_{(T_{11}, T_{12}=T-T_{11})}.\end{aligned}$$

where $JMU_{T_{1r}} = \frac{\partial U}{\partial Z_1}\frac{\partial Z_1}{\partial T_{1r}} + \frac{\partial U}{\partial Z_2}\frac{\partial Z_2}{\partial T_{1r}}, r = 1, 2$ denotes joint marginal utility of the use of time $T_{1r}$. Under the plausible assumption

$$JMU_{T_{11}} < JMU_{T_{12}}, \text{ for } T_{11} = T, \tag{2.11}$$

the boundary solution in the case that the budget constraint is binding must be

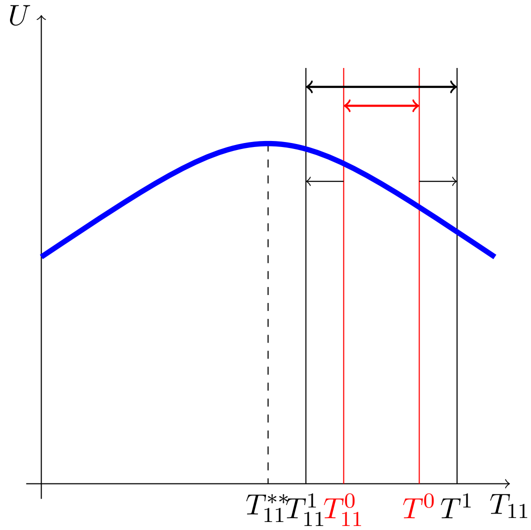


Figure 2.2: Retirement age paradox, typical solution when budget constraint binds

$$T_{11}^{*} = T_{11}^{min} = \frac{G - V - w_{12}T}{w_{11} - w_{12}}. \tag{2.12}$$

If it is further assumed

$$JMU_{T_{11}} > JMU_{T_{12}}, \text{ for } T_{11} = 0, \tag{2.13}$$

which is also sensible, and the function $JMU_{T_{11}} - JMU_{T_{12}}|_{(T_{11}, T_{12}=T-T_{11})}$ is decreasing with respect to $T_{11}$; figure 2.2 represents the situation in this case. Under the assumptions above, the utility as a function of the working years ($T_{11}$) presents an inverted-$U$ shape. Notice that $T_{11}^{**}$ is the unconstrained choice of the individual in this case, but it cannot be implemented due to the budget constraint.

This case has interesting implications in economic terms: the individual must spend more time working ($T_{11}^{min}$) than she likes the most ($T_{11}^{**}$ in figure 2.2) in order to meet her economic standards –represented by $G$– given the work life ($T_1 = T$) by law.

Now assume that the government is considering to increase the retirement age with, say, the purpose of making more sustainable the pension system. Such policy actually increases the potential work life $T$ from $T^0$ to $T^1$ (see figure 2.2). However, it can be observed in figure 2.2 that the overall effect of such policy consists of a reduction in actual working time, and the individual reduces working time from $T_{11}^0$ to $T_{11}^1$. This reduction corresponds to the following expression, obtained from (2.12):

$$\frac{\partial T_{11}^*}{\partial T} = \frac{-w_{12}}{w_{11} - w_{12}}. \tag{2.14}$$

Since we have assumed that $w_{11}, w_{12} > 0$ satisfy $w_{11} > w_{12}$, it follows from (2.14) that $\dfrac{\partial T_{11}^*}{\partial T} < 0$.

Therefore, a policy maker may observe a reaction from individuals consisting of reducing their working time, which would be potentially harmful in terms of public policy (alleviating public expenditure), and the opposite effect that was expected *a priori*.

In order to illustrate the model analysis with some numerics, we next do some numerical analysis using concrete functions $Z$ and $U$, such that conditions (2.11), (2.12) and (2.13) hold. We have considered the following parametric expressions:

$$U(Z_1, Z_2) = a_0 + a_1 ln(Z_1) + a_2 ln(Z_2), \tag{2.15}$$

$$Z_1 = b_{11} ln(1 + T_{11}) + b_{12} ln(1 + T - T_{11}), \tag{2.16}$$

$$Z_2 = b_{21} ln(1 + T_{11}) + b_{22} ln(1 + T - T_{11}). \tag{2.17}$$

The particular values of the parameters are shown in tables 2.1 and 2.2. We consider an increase in the retirement age of two years, from $T = 49$ to

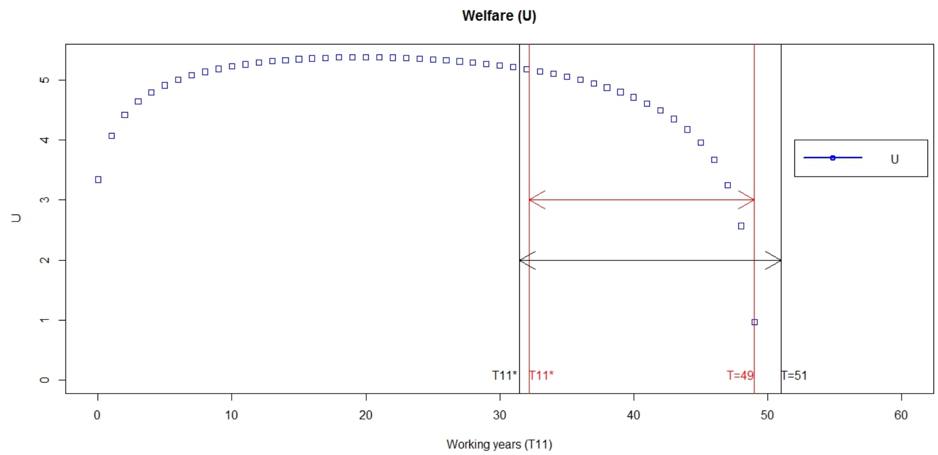


Figure 2.3: Numerical analysis of model in (2.10) representing the retirement age increase paradox.

Table 2.1: Parametrical values for the inputs

| Inputs | | | | | |
|---|---|---|---|---|---|
| $T$ | $w_{11}$ | $w_{12}$ | $G$ | $g$ | $V$ |
| $49 \to 51$ | 30000 | 8000 | 1100000 | 22500 | 0 |

Table 2.2: Parametrical values for utility function and satisfactions functions

| Utility | | | Satisfactions | | | |
|---|---|---|---|---|---|---|
| $a_0$ | $a_1$ | $a_2$ | $b_{11}$ | $b_{12}$ | $b_{21}$ | $b_{22}$ |
| 0 | 2 | 3 | 0.6 | 0.3 | 0.2 | 0.7 |

$T = 51$; this corresponds to an increase in the retirement age from 65 to 67 years. Running the model for this parametric values, we obtain the results displayed in figures 2.3 and 2.5.

As expected the numerical analysis in figure 2.3 matches the theory described earlier.

This model illustrates in a very simple manner a paradox. However, it may be argued that keeping $G$ constant when $T$ increases is not realistic. To account for that, consider a variation of the model in which $G = gT$, where $g$ is the average expenditure per year in the working lifetime period. It must be satisfied that $w_{11} > g$. This refinement generates the following model:

$$\begin{cases} \max\limits_{T_{11}} & U(Z_1(T_{11}, T - T_{11}), Z_2(T_{11}, T - T_{11})), \\ s.t. & gT \leq (w_{11} - w_{12})T_{11} + w_{12}T + V, \\ & 0 \leq T_{11} \leq T. \end{cases} \tag{2.18}$$

Under the assumptions for $U$ and $Z_i$ considered above, the boundary solution in this case is given by

$$T_{11}^* = \frac{(g - w_{12})T - V}{w_{11} - w_{12}}. \tag{2.19}$$

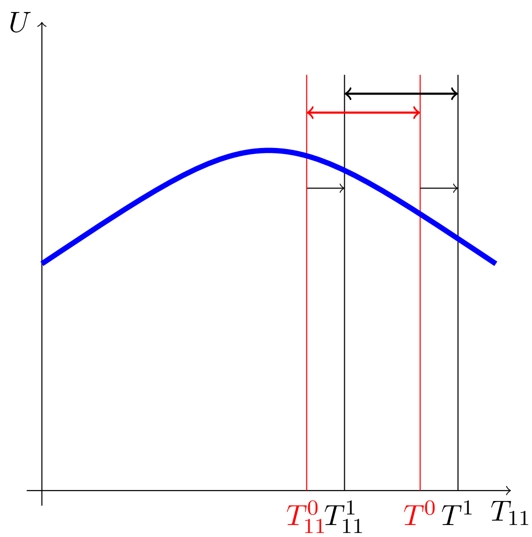


Figure 2.4: Retirement age insufficiency, solution extended case

Since

$$\frac{\partial T_{11}^*}{\partial T} = \frac{g - w_{12}}{w_{11} - w_{12}}, \tag{2.20}$$

and $w_{11} > w_{12}$, the paradox also occurs if $g < w_{12}$, which is not expected to hold in general. However, if $g \geq w_{12}$ a weaker version of the paradox still may hold. We have that individuals will decide to increase their working time by less time than the time increase in the retirement age.

This is because

$$\frac{\partial T_{11}^*}{\partial T} = \frac{g - w_{12}}{w_{11} - w_{12}} \geq 0, \tag{2.21}$$

and $w_{11} > g \geq w_{12} > 0$ imply that $g - w_{12} < w_{11} - w_{12}$, so that $\frac{\partial T_{11}^*}{\partial T} < 1$, and then $\Delta T_{11}^* < \Delta T$.

Thus, the increase in the retirement age will not be fully covered by working time.

It follows from our analysis that the public policy may not be as effective as expected. This conclusion is particularly interesting in a situation in which the retirement age would be increased by law to account for the increase in life expectancy.

We illustrate this extended model numerically using the same functions in (2.15), (2.16) and (2.17). Also the values for the parameters are those given before, except from $G$. Now we have $G = gT$, with $g = 22500$. We consider an increase in retirement age of two years, as above. Our results are illustrated in figure 2.5:

We observe, again, that our theoretical analysis illustrated in figure 2.4 is replicated by the numerical analysis showed in figure 2.5. That is, an increase in the retirement age is not transferred completely in working time by the individual rational decision. So, for instance, if the retirement age is extended in 24 months, the individual would increase her working time only by about 16 months, an incomplete proportion of the increase in the retirement age. Under the assumptions above, such an expenditure level

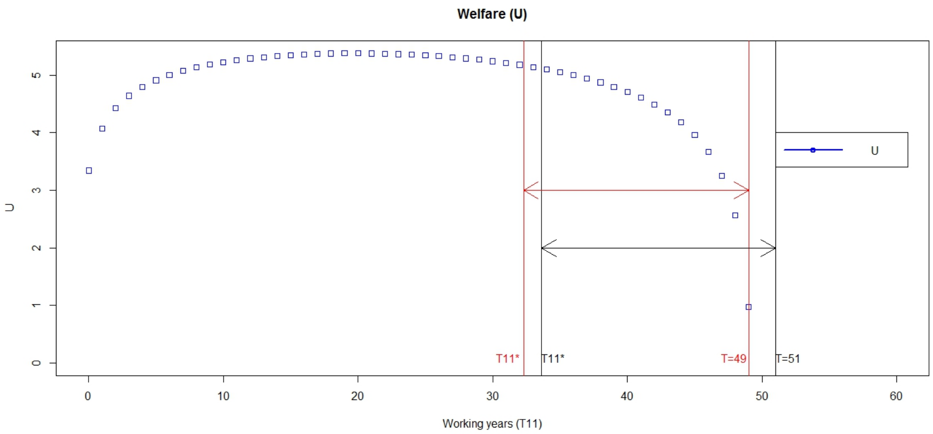


Figure 2.5: Numerical analysis of model in (2.18) representing the increasing the retirement age insufficiency

actually obliges her to work more than she would wish.

Table 2.3: Solutions, working time ($T^*_{11}$) for a policy consisting of an increase in retirement age of two years ($T = 49 \rightarrow T = 51$)

| Case with $G$ | | Case with $G = gT$ | |
|---|---|---|---|
| $T^*_{11}$ | $\Delta T^*_{11}$ vs. $\Delta T$ | $T^*_{11}$ | $\Delta T^*_{11}$ vs. $\Delta T$ |
| $32.18 \rightarrow 31.45$ | $-0.73 < +2$ | $32.29 \rightarrow 33.61$ | $+1.32 < +2$ |

**Some remarks**

It must be noticed that this simple illustration shows an example in which joint production is an unavoidable phenomena. We do it by reducing the extended model in (2.7) to a case in which there is only one type of time (work life) during which an individual must decide whether to work or not. This simple way of expressing the extended model in (2.7) does not highlight very much the central role attributed to the matrix defined in (2.6), which in this case is a matrix with dimension $(1 \times 2)$.

However, it is easy to think of further examples with a simple matrix of time inputs with $(2 \times 2)$ dimension; for example a week day for, say, a researcher. We can define two types of time, let us say, working hours detailed in the researcher's job contract from 8.00h to 17.00h ($T_1 = 9h$) and the remaining hours of the day ($T_2 = 15h$). This researcher must decide how to allocate her time either working $T_{l1}$ or not working $T_{l2}$, for all types of time denoted with $l = 1, 2$. Therefore, the matrix of time inputs would be

$$\Im_{(2\times 2)} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix}, \tag{2.22}$$

The sum of the working and non-working time in the usual working hours must fulfil $T_{11} + T_{12} = T_1 = 9h$, and similarly for the remaining time of the week day. Job satisfaction and personal satisfaction are still the wants here, and notice that these are produced unavoidably jointly. The matrix (2.22) defined above indicates the work life balance in a week day for this researcher, who is assumed to maximize her welfare subject to both budget and time constraints, as in the example concerning retirement, which we analysed in depth above. For this example, a similar analysis could have been developed, consisting of analysing the effect of increasing the usual working hours $T_1$ on actual working time $T_{11}$ for this individual within usual working hours signed by contract, $T_1$.

### Conclusions from this simple illustration

Whenever money imposes a problem/constraint in a lifetime perspective, policy decisions in favour to increasing retirement age may lead to a reduction of working years or, at most, to an insufficient increase on working time. Therefore, although the result of this policy may result in an increase in individual welfare, such decision may not fulfil the public goals expected by policy makers for, say, fiscal reasons. Both theoretical predictions and numerical analysis confirm this assertion in a very simple model case.

# Chapter 3

# Time microeconomics and choice overload[1]

*With every lost hour, a part of life perishes.*

(Gottfried Wilhelm Leibniz)

The extended model presented in chapter 2 serves as a general framework in order to analyse more concrete problems, which is the goal of this chapter. We start with an introduction to the paradox of choice and other choice overload situations in section 3.1. This is followed in section 3.2 by a reduction of the extended theory to a case in which we will just consider a pure time use decision among three uses. We set up a time microeconomic model based on the choice of one product with many different options in the market and we show its main features and we analyse its general solutions in section 3.3. The model admits different solutions which are described by different consumer profiles. We discuss the main cases in section 3.4, offering an intuitive and complementary graphical analysis illustrating the differences among all profiles. This discussion suggests the possibility of choice overload situations. Last, in section 3.5 we provide formal conditions under which the model produces choice overload and we prove the corresponding mathematical results.

[1] The main implications of this chapter were presented at the International Conference on "Mathematical Modelling in Engineering and Human Behaviour 2012" September 4-7, 2012, Instituto Universitario de Matemática Multidisciplinar, Polytechnic City of Innovation in Valencia, Spain.

## 3.1 Introduction to choice overload problems

There is an intrinsic assumption for any model in economics which basically links more choice to more welfare. However, in the last decade some studies have suggested a counter-intuitive fact, according to the consumer culture in western societies. Iyengar and Lepper (2000) and Schwartz (2000, 2004, 2005, 2006) comment on and illustrate empirically this phenomenon which has been named choice overload or the paradox of choice.

Barry Schwartz suggested the logic behind the underpinnings of the paradox of choice by questioning the following syllogism:

*More freedom means more welfare*

*More choice means more freedom*

*Therefore, more choice means more welfare*

Schwartz (2000) talks about the existence of what he calls the tyranny of freedom, which in turn is directly translated as more freedom of choice; at the end of the day, that leads to a tyranny of choice which challenges the syllogism above. Therefore, more freedom of choice –or simply, more choice– will actually not lead to more welfare. The potential implications of this idea in microeconomic terms are of high interest. Moreover, Schwartz (2000) claims to the existence of a paralysis effect when decision makers face a choice with high number of options, which in turn force them to quit the act of choosing.

Although this could seem a subjective reasoning from the field of psychology, it finds empirical support from some experiments or field studies. Iyengar and Lepper (2000) triggered not only this idea, but also some collateral findings. This paper puts on the table three experiments that support the idea that more choice does not imply more welfare.

Schwartz (2005), a book completely devoted to the paradox of choice, is written for the general public and provides lots of insights some of which we formalize in our time microeconomic model, to be discussed later in this chapter. Mogilner et al. (2008) mainly analyses the so called categorization effect; this idea revolves around making the decision process easier by grouping similar choice options and establishing categories so that the decision makers differentiate more easily the right choice option.

Besides psychology, other social sciences have paid attention to the issue of choice overload, which can be considered an interdisciplinary problem. Reutskaja and Hogarth (2009) and Reutskaja (2008) give other examples. They start by assuming a utility such that its shape is an inverted-U with respect to the number of options in a choice set. They provide some heuristic background justifying why this assumption is plausible. This is shown in figure 3.1 and pictures that can be seen in figure 3.2.

**Table 1. Benefits and Costs of Choice as a Function of Number of Alternatives.**

| Factors | Benefits | Costs |
|---|---|---|
| Situational | | |
| Time | | Increasing (linear) |
| Economic | Increasing (decreasing rate) | |
| Psychological | | |
| Cognitive | | Increasing |
| Psychic | Increasing (decreasing rate) | Increasing |

Figure 3.1: Table from Reutskajan and Hoghart (2009)

Essentially, these contributions show experimental validation to the assumption of an inverted-U shape for the utility with the number of options. However, they claim that further research is needed to incorporate for example, biological measures. They address that issue in a subsequent paper (Reutskaja et al., 2011), where they try to replicate the consumer problem when she goes to the supermarket. This contribution is out of our approach, since it concentrates more in understanding the effects of increasing the number of choice options through some biological measurement, as the eye tracking device they use in their experiment.

Another interesting contribution is Dar-Nimrod et al. (2009), which pro-

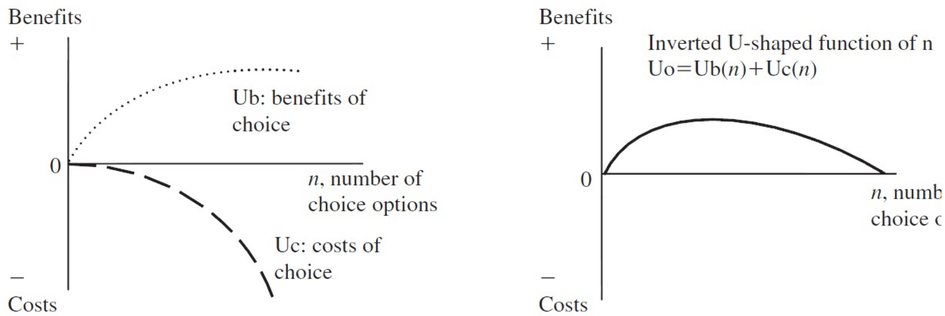


Figure 3.2: Pictures from Reutskajan and Hoghart (2009)

poses a distinction between maximizers and satisficers as different profiles of seekers. Once that is done, they present an study in which they show how more choice is not good for individual satisfaction, especially for maximizers. This is explained in their conclusion as what they call the *Maximization Paradox*, which we reproduce here:

> *"Choice is highly valued in society. More options from which to choose are perceived as better than are fewer options because, logically, the larger set is more likely to yield a desirable option (Iyengar and Lepper, 2000; Schwartz, 2004). Yet some people, particularly maximizers, suffer adverse consequences from the promise offered by the larger set of alternatives. Identifying this paradox is an important step toward helping people deal with the vast arrays of options they can choose to face or avoid on a daily basis"* (Dar-Nimrod, Rawn, Lehman and Schwartz, 2009).

Chabris et al. (2009) demonstrate using a theoretical model that people spend more time on decision making when the options are more similar.

Some studies have questioned the general validity of choice overload. Scheibehenne et al. (2010) survey and test all the empirical research up to the date, and they do not find robust conclusions in favour of choice overload; nevertheless they do not discard it. They point to environmental problems in the experiments and argue that their own results may be the key to explain why it has not been possible to replicate many of the experiments which evidenced choice overload. In fact, lack of replicability is an important general criticism for all this experimental research.

Markus and Schwartz (2010) and Grant and Schwartz (2011) are further theoretical contributions from the field of marketing and consumer behaviour.

More recently, economics has also paid attention to problems related to choice overload. For example Iyengar and Kamenica (2010) shows that increasing the number of options not only reduces the participation in the market and reduces the welfare of consumers, but also affects to the type of options that are chosen. In addition, Kamenica et al. (2011), explores how*"[m]odern information processing technologies are allowing sellers to know increasingly more about their consumers' purchasing behaviour"*; their approach is based on how the asymmetry of information[2] is getting in favour of sellers, who may react changing their price structure to help consumers to be better shoppers. A recent publication in an economic journal is Caplin et al. (2012) and offers a search model that deals with everyday situations of choice in which, as we do not see all options, we might be missing the best. However, we get a good enough option since sequentially we update our reservation utility in order to stop. An interesting insight in the paper is the possibility of the existence of a reservation time. If there is such a thing, reaching the reservation time would provide a sensible stopping rule in a decision making process. The experiments carried out in the paper, however, reject that possibility.

## 3.2 Reducing the extended model

In this chapter we reduce the theory in chapter 2 to a simpler setting which turns out to be useful in order to model, analyse and solve for some open questions related with choice overload.

We consider an individual who has three wants or commodities: satisfaction of shopping, personal satisfaction and job satisfaction, respectively: $Z_1, Z_2, Z_3$. In this setting we have just one type of time and three different uses of time: shopping time ($T_b$) spent on searching and selecting some product within a choice set of similar versions of the product –which produces shopping satisfaction–, free time ($T_c$) –generates personal satisfaction– and

[2]Asymmetry of information in general, where time use can be regarded

labour time ($T_l$) –determines job satisfaction–. If we think in terms of the notation in (2.7), we have, so far, $m = 3$, $p = 1$ and $r = 3$.

The expenditure in the problem, modelled by the $G$ function, must also be specified. The problem refers to a choice of a good within a choice set of similar goods which have different prices in the market. Therefore, there is one category ($q = 1$) and a number of given options $N$ among which we have to chose. The expenditure in the budget set is a function of the number of alternatives:

$$G = G(N). \tag{3.1}$$

Therefore, a reduced version of the general problem of chapter 2 in (2.7) is so obtained.

To summarize, the model in (2.7) is reduced here to a situation with $m = 3, p = 1, r = 3, n = N, q = 1$, and $G$ being a function of $N$ as in (3.1). The non-negativity constraints on the uses of time might be replaced by different minimum time use requirements.

## 3.3 A time microeconomic model

This model considers an individual who has a limited amount $T > 0$ of time to be spent in three different uses: working to get income $T_l$, shopping time $T_b$ to search and decide what to buy among similar alternatives offered in the market, and free time $T_c$ to enjoy himself. Therefore, this individual faces the following time constraint:

$$T_b + T_c + T_l = T. \tag{3.2}$$

Let

$$\Delta = \{\vec{T} = (T_c, T_b, T_l) : T_c, T_b, T_l \geq 0, T_c + T_b + T_l = T\} \tag{3.3}$$

be the 3-D time simplex represented in figure 3.3.

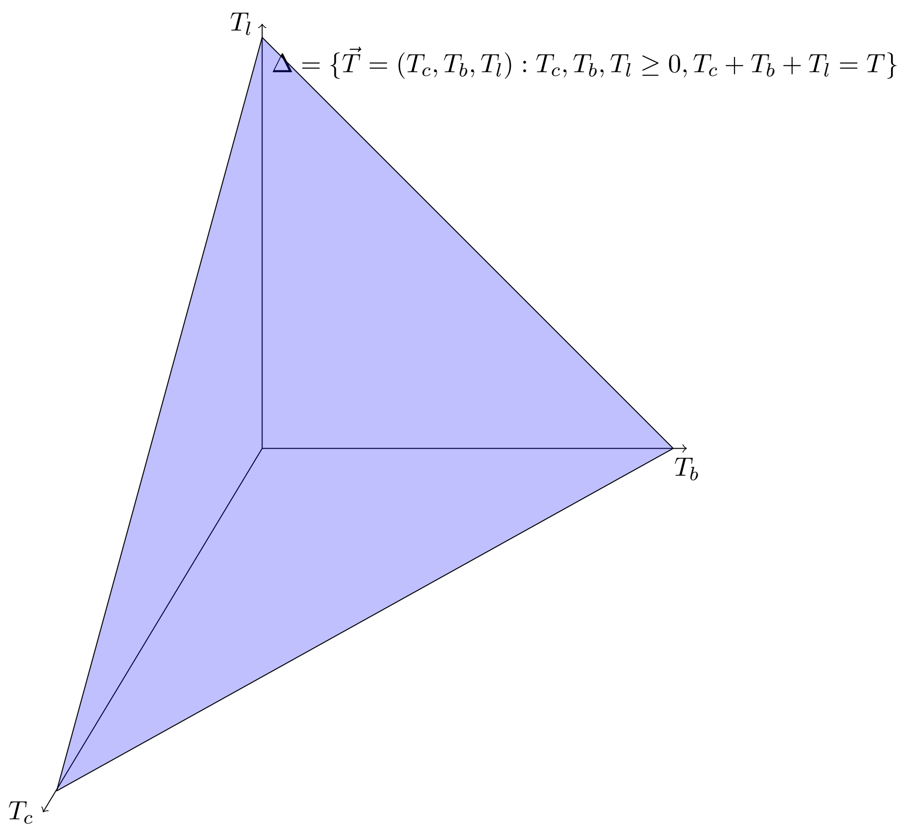


Figure 3.3: The time simplex 3D

In order to search, compare and decide what product to choose among the alternatives in the market, there is a minimum amount of time to be spent. Such minimum time is described by a function $\tau(N)$ of the total number $N$ of options or alternatives of a similar product in the market. The individual thus faces the following time constraint:

$$T_b \geq \tau(N), \tag{3.4}$$

where typically $\tau(N)$ is non-decreasing with the number of options $N$.

Moreover, the expenditure $G$ of this individual cannot be larger than the total income obtained by working plus other available income $V$. The individual faces the following budget constraint:

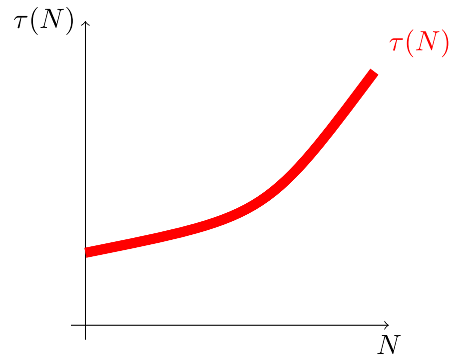


Figure 3.4: Shopping time floor ($\tau(N)$) as the number $N$ of options in the market increases.

$$G(N) \leq wT_l + V, \tag{3.5}$$

where typically $G(N)$ is non-increasing with the number of options $N$, $G(N) > V$ and it approaches some value $\overline{G}$ as $N$ increases (see figure 3.5).

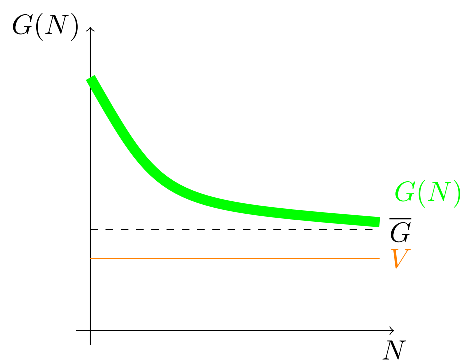


Figure 3.5: Expenditure ($G(N)$) as the number $N$ of market options increases.

Decisions on time uses produce several satisfactions. In this model, time searching in the market produces certain satisfaction of shopping $Z_1$, free time generates personal satisfaction $Z_2$, while working time has an impact on job satisfaction $Z_3$.

Notice joint production is not considered in the model; for simplicity here each use of time serves to each one of the satisfactions. This will suffice for our purpose. Hence, the satisfactions are generated according to:

$$Z_1 = f_1(T_b),$$

$$Z_2 = f_2(T_c),$$

$$Z_3 = f_3(T_l).$$

Individual welfare is determined by the satisfaction levels described above, as an utilitarian assessment. This is modelled by the following expression:

$$U\left(Z_1, Z_2, Z_3\right). \tag{3.6}$$

The utility increases as each of these satisfactions increase, that is

**Assumption** [ $U$**1** ]

$$\frac{\partial U}{\partial Z_m} > 0,\ for\ m = 1, 2, 3. \tag{3.7}$$

This approach follows the line started by Ironmonger (1972) and Becker (1965), where the arguments of the utility function are not directly inputs of goods or time, but what they name as wants or commodities, respectively.

The individual seeks to maximize her overall welfare in (3.6), subject to the constraints defined in (3.5), (3.4), (3.2) and non-negativity constraints on $T_c$ and $T_l$. Hence, the model is finally set as follows:

$$\begin{cases} \max\limits_{T_b,T_c,T_l} & U = U(Z_1(T_b), Z_2(T_c), Z_3(T_l)), \\ s.t. & G(N) \leq wT_l + V, \\ & T_b + T_c + T_l = T, \\ & T_b \geq \tau(N), \\ & T_c \geq 0, \\ & T_l \geq 0. \end{cases} \tag{3.8}$$

The feasible set of this problem, –which we will refer to as the $N$-choice set–, is mathematically defined by:

$$\Omega(N) = \left\{\vec{T} \in \Delta : T_b \geq \tau(N), G(N) \leq wT_l + V\right\}.$$

The $N$-choice set is represented graphically in figure 3.6.

Notice that our model is not a search model in the sense of Stigler (1961). We do not consider a dynamic or inter-temporal setting, as in search theory.

Without loss of generality, the model in (4.1) can be reduced to the following 2D problem,

$$\begin{cases} \max\limits_{T_b,T_c} & U = U(Z_1(T_b), Z_2(T_c), Z_3(T - T_b - T_c)), \\ s.t. & G(N) \leq w(T - T_b - T_c) + V, \\ & T_b + T_c \leq T, \\ & T_b \geq \tau(N), \\ & T_c \geq 0, \end{cases} \tag{3.9}$$

where working time $T_l$ is removed explicitly as a choice variable, its non-negativity is guaranteed by the second constraint in (3.9) if $G(N) > V$, and it can be calculated as a residual.

Moreover, projecting both the time simplex and the 3D-choice set into the plane, we obtain the 2D-time simplex and the *time triangle* as it is shown in figures 3.7 and 3.8, respectively. Mathematically, we define the 2D-time simplex as

$$\Delta_2 = \{(T_c, T_b) : T_c, T_b \geq 0, T_c + T_b \leq T\}.$$

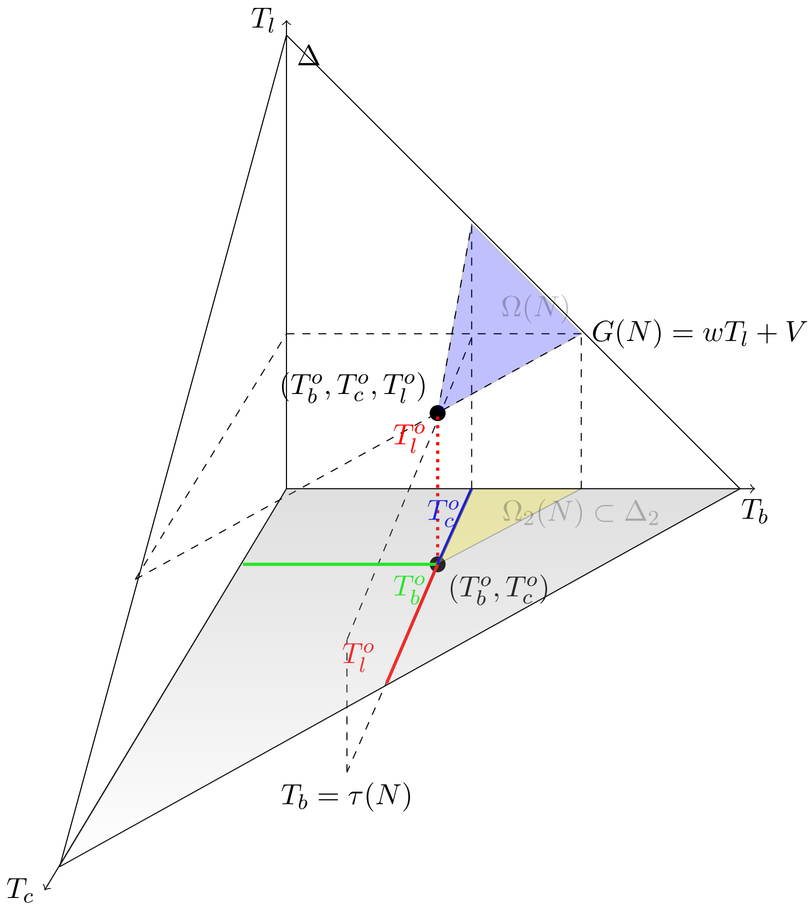


Figure 3.6: The 3D-choice set $\Omega(N)$ (blue shaded) contained in the 3D-time simplex $\Delta$. Projections in 2D yield the choice set $\Omega_2(N)$ (the *time triangle*, yellow shaded) contained in the time simplex $\Delta_2(N)$ (grey shaded)

Similarly, the time triangle is given by the 2D-choice set

$$\Omega_2(N) = \{(T_c, T_b) \in \Delta_2 : T_c \geq 0, T_b \geq \tau(N), T_c + T_b \leq T\}. \tag{3.10}$$

We will use the time triangle in figure 3.8 to illustrate the analysis of the model.

The Lagrangian associated to the problem in (3.9) is

$$\begin{aligned} L(T_b, T_c, \lambda, \delta, \mu, \varepsilon_c) = \; & U(Z_1(T_b), Z_2(T_c), Z_3(T - T_b - T_c)) \\ & -\lambda(G(N) - w(T - T_b - T_c) - V) \\ & -\delta(T_b + T_c - T) \\ & -\mu(\tau(N) - T_b) \\ & +\varepsilon_c T_c. \end{aligned} \tag{3.11}$$

The first order necessary conditions for a solution of the problem (3.9) consists of the following set of conditions:

$$\frac{\partial U}{\partial Z_1}\frac{\partial Z_1}{\partial T_b} - \frac{\partial U}{\partial Z_3}\frac{\partial Z_3}{\partial T_l} - \lambda w - \delta + \mu = 0, \tag{3.12}$$

$$\frac{\partial U}{\partial Z_2}\frac{\partial Z_2}{\partial T_c} - \frac{\partial U}{\partial Z_3}\frac{\partial Z_3}{\partial T_l} - \lambda w - \delta + \varepsilon_c = 0, \tag{3.13}$$

$$G(N) = w(T - T_b - T_c) + V, \lambda \geq 0, \text{ or} \tag{3.14}$$
$$G(N) < w(T - T_b - T_c) + V, \lambda = 0, \tag{3.15}$$

$$\begin{aligned} T_b + T_c = T, \delta \geq 0, & \text{ or} \\ T_b + T_c < T, \delta = 0, & \end{aligned}$$

$$T_b = \tau(N), \mu \geq 0, \text{ or} \tag{3.16}$$
$$T_b > \tau(N), \mu = 0, \tag{3.17}$$

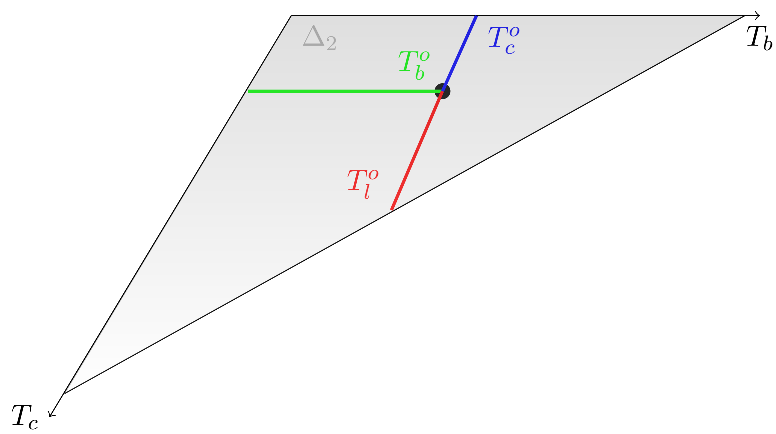


Figure 3.7: The 2D-time simplex

$$T_c = 0, \varepsilon_c \geq 0, \text{ or}$$
$$T_c > 0, \varepsilon_c = 0.$$

The shadow value in terms of utility of an extra unit of money is represented by $\lambda$; thus $\lambda w$ is the subjective value of an extra unit of total time $T$, i.e. the 25-th hour of a day, since the only valuation of time at market prices is $w$. The shadow value of one additional time unit of the minimum shopping time $\tau(N)$ corresponds to $\mu$.

The model admits $(2^4)$ possible cases for the solution. That depends upon the different combinations among binding or non-binding constraints. However, if we focus on solutions for which the three uses of time are positive, cases for discussion are reduced to four $(2^2)$; these four cases are generated from combinations of conditions (3.14) and (3.15) with (3.16) and (3.17).

Let us explain in more detail the time triangle introduced in 3.7. The 2-dimensional projection of the $N$-choice set $\Omega(N)$ (figure 3.6) generates the time triangle $\Omega_2(N)$. How is the time triangle affected by changes in $G(N)$ or in $\tau(N)$? It is clear that a reduction in $G(N)$ –e.g. an increase in the market options from $N_1$ to $N_2$ with $N_1 < N_2$– moves the plane $G(N) = wT_l + V$ towards the $T_c - T_b$ plane (see $\Omega_2(N)$) in figure 3.6; this change shifts upwards the green line in the time triangle which delimits the feasible set in

$\Omega_2(N_1)$ (yellow shaded in figure 3.8). An increase in $\tau(N)$ –e.g. due to an increase from $N_1$ to $N_2$, $N_1 < N_2$– shifts the plane $T_b = \tau(N)$ to the right in figure 3.6; as a result, the red line in the time triangle in $\Omega_2(N_1)$ (yellow shaded) shifts upwards in figure 3.8. The overall effect in the time triangle within the 2D-time simplex is represented by the change from the yellow area ($\Omega_2(N_1)$) to the resulting purple area ($\Omega_2(N_2)$).

Notice that both a wage raise (increase in $w$) and an increase in savings $V$ would imply a similar shift of $\Omega_2$ in the budget constraint in figure 3.8.

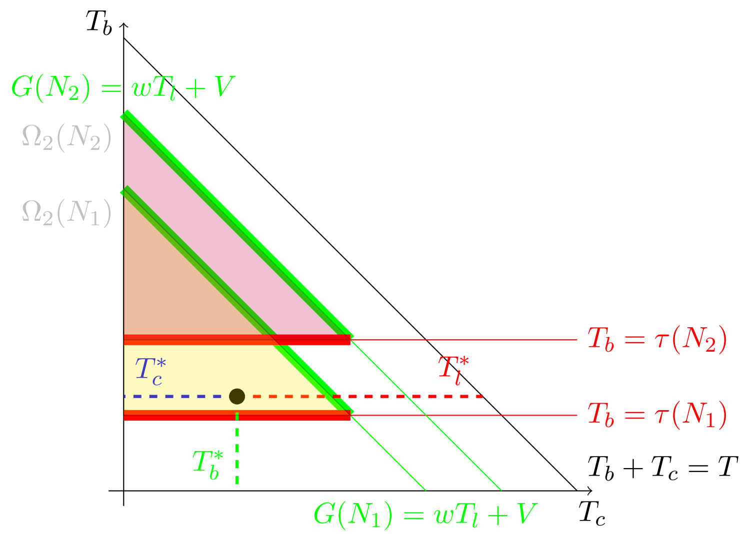


Figure 3.8: The time triangle changes (from $\Omega_2(N_1)$ –yelow shaded– to $\Omega_2(N_2)$, purple shaded) due to a reduction in $G(N)$ and an increase in $\tau(N)$ as a consequence of increasing the number of market options $N$.

The following general assumption will be used for the sequel:

**Assumption** [ $G - V$ ] (Savings are not enough) Consumer expenditure is larger than non-labour income, that is

$$G(N) > V. \tag{3.18}$$

Notice that (3.18) implies working time must be positive, since consumers cannot meet their expenditure level without working. This sets $\delta = 0$.

## 3.4 Discussion of several cases

The model described in the previous section generates multiple cases ($2^4$). We discuss here different cases in which the $N$-choice set defined in (3.10) is not empty, so the time triangle is as represented in figure 3.8.

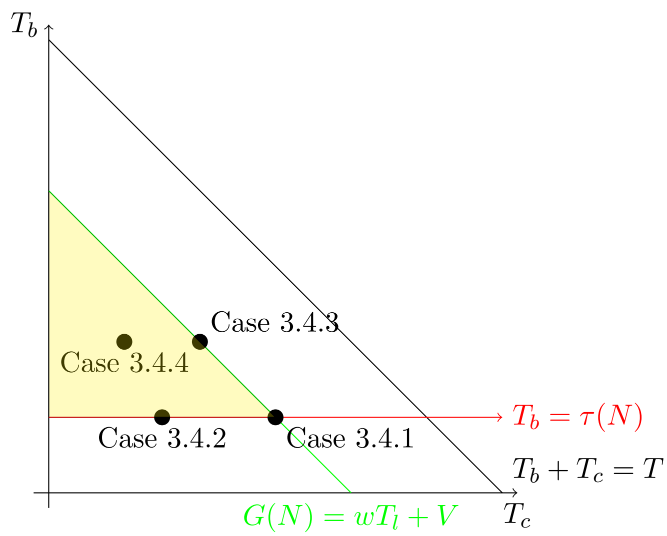


Figure 3.9: Possible cases for the solution of the problem, in the time triangle

The four cases that we describe can be related to different profiles of consumers. Some can be described as ordinary consumers (case 3.4.1), others as workaholics, shopping lovers, or as unconstrained consumers (cases 3.4.2, 3.4.3, 3.4.4, respectively). We understand here that an *ordinary consumer* is an individual who spends all the money she gets and invests the least possible time shopping. A *workaholic* is a person who also spends the least possible time shopping, but prefers to work more hours instead of enjoying more free time; this implies that she gets more income than the amount needed for the consumption. A *shopping lover* is a person who spends all her income and prefers to dedicate more hours than the minimum needed for shopping time. Lastly, an *unconstrained consumer* is a consumer whose time allocation is not affected by budget or time constraints.

We describe below each case above separately. For each case we show geometrical solutions in the time triangle; and we provide some discussion of the behaviour of the optimal solutions and welfare when the parameters of the model vary and, especially, when the number of options increases.

For the rest of the discussion, recall that $T_c > 0$ so that $\varepsilon_c = 0$ in (3.12)-(3.18). Also, assumption (3.18) implies that $T_l > 0$ and then $\delta = 0$ from (3.16). The time floor constraint (3.4) ensures $T_b > 0$. All cases discussed below correspond to positive time uses $T_c > 0, T_l > 0, T_b > 0$. We will also assume during section 3.4 that

$$\frac{\partial Z_2}{\partial T_c} > 0.$$

This seem a plausible assumption in general.

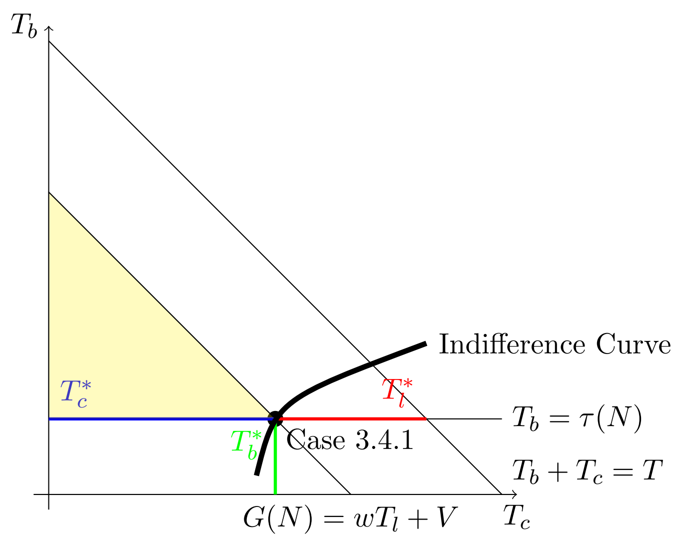


Figure 3.10: Case of ordinary consumers, described in 3.4.1

### 3.4.1 Case of ordinary consumers

This corresponds to the case in which the solution is located at the lower right vertex of the time triangle (see figure 3.10) so that both the budget and time floor constraints are binding. In this situation the consumer works up to the minimum level that is needed to meet her budget constraint. Also, the consumer spends the least necessary time searching and deciding what to purchase.

In sum, an ordinary consumer is an individual who must work to buy what she wants, but spends just the least possible time shopping and working in order to have the most possible free time available.

The optimal solution for the case of ordinary consumers must satisfy the following conditions:

$$T_b^* = \tau(N), \tag{3.19}$$

$$T_c^* = T - \tau(N) - \frac{G(N) - V}{w}, \tag{3.20}$$

$$T_l^* = T - T_b^* - T_c^* = \frac{G(N) - V}{w}, \tag{3.21}$$

$$\lambda^* = \frac{1}{w}\left(\frac{\partial U}{\partial Z_2}\frac{\partial Z_2}{\partial T_c} - \frac{\partial U}{\partial Z_3}\frac{\partial Z_3}{\partial T_l}\right) \geq 0, \tag{3.22}$$

$$\mu^* = \frac{\partial U}{\partial Z_2}\frac{\partial Z_2}{\partial T_c}(\vec{T}^*) - \frac{\partial U}{\partial Z_1}\frac{\partial Z_1}{\partial T_b}(\vec{T}^*) \geq 0, \tag{3.23}$$

$$\delta^* = 0 = \varepsilon_c^*,$$

where all derivatives are evaluated at the optimum.

The optimal allocation of free time, shopping time and working time $(T_c^*, T_b^*, T_l^*)$ is represented in figure 3.10. In this case, both constraints bind.

Notice that the solution of the ordinary consumer is obtained under some simple assumptions on the individual's preferences, namely $\dfrac{\partial U}{\partial Z_i} > 0$ and

$$\frac{\partial Z_1}{\partial T_b} < 0, \frac{\partial Z_2}{\partial T_c} > 0, \frac{\partial Z_3}{\partial T_l} < 0.$$

Indeed, in that case, we have from (3.22) that $\lambda^* > 0$ so that (3.14) applies and the budget constraint is binding. Also, we have from (3.23) that $\mu^* > 0$ and then (3.16) implies that the time floor constraint is active. This corresponds to the case that the solution is located at the lower right vertex of the time triangle, that is, the case of ordinary consumers.

Since solutions are explicit, their behaviour with respect to variations in the parameters can be easily obtained:

$$\frac{\partial T_b^*}{\partial w} = 0, \tag{3.24}$$

$$\frac{\partial T_c^*}{\partial w} = \frac{G(N) - V}{w^2} > 0, \tag{3.25}$$

$$\frac{\partial T_l^*}{\partial w} = -\frac{G(N) - V}{w^2} < 0, \tag{3.26}$$

$$\frac{\partial T_b^*}{\partial V} = 0, \tag{3.27}$$

$$\frac{\partial T_c^*}{\partial V} = \frac{1}{w} > 0, \tag{3.28}$$

$$\frac{\partial T_l^*}{\partial V} = -\frac{1}{w} < 0, \tag{3.29}$$

$$\frac{\partial T_b^*}{\partial T} = 0, \tag{3.30}$$

$$\frac{\partial T_c^*}{\partial T} = 1, \tag{3.31}$$

$$\frac{\partial T_l^*}{\partial T} = 0. \tag{3.32}$$

Each time use is sensitive to changes in parameters such as the wage rate, savings and total time available ($w$, $V$ and $T$ respectively). These changes are described by expressions (3.24) to (3.32). A wage raise presents a clear impact: working time will fall and will be switched by an increase in free time; shopping time will not be affected. Similarly, an increase in savings would have the same impact than a wage raise. Lastly, if an ordinary consumer could have more total time $T$, the only impact on optimal solutions would be an increase in both working and free time.

The value function of the problem –denoted from now on as $W = U^*(T, w, V, N) = U\left(\vec{Z}(\vec{T}^*)\right)$– is obtained by inserting the optimal solutions into the objective function. Changes in the optimal welfare with respect to some parameters of the problem can be easily obtained from (3.11) by means of the envelope theorem for Lagrangian problems as follows:

$$\frac{\partial W}{\partial w} = \left.\frac{\partial L}{\partial w}\right|_* = \lambda^* \frac{G(N) - V}{w} \geq 0,$$

$$\frac{\partial W}{\partial V} = \left.\frac{\partial L}{\partial V}\right|_* = \lambda^* \geq 0,$$

$$\frac{\partial W}{\partial T} = \left.\frac{\partial L}{\partial T}\right|_* = \frac{\partial U}{\partial Z_3}\frac{\partial Z_3}{\partial T_l} + \lambda^* w > 0.$$

It follows from these expressions that either a wage raise or an increase in savings increases her welfare since her shadow value of money is positive.

Also, under the assumption that $\dfrac{\partial Z_2}{\partial T_c} > 0$, an increase in $T$ (i.e. the 25-th hour of a day) implies that she would be better off, since she would enjoy more free time. The responses of the solution to changes in the parameters match our *a priori* intuition, under sensible assumptions. This enhances the plausibility of the proposed model.

### Choice overload behaviour for ordinary consumers

Each time use and the associated shadow values are sensitive to changes in the number of market options ($N$). An increase in the number of market options has an uncertain impact on free time and shadow values, but not on shopping time (which increases) and working time (which decreases). Although below we consider two (extreme) cases whose analysis is straightforward, in general the impact on free time and shadow values is not obvious[3].

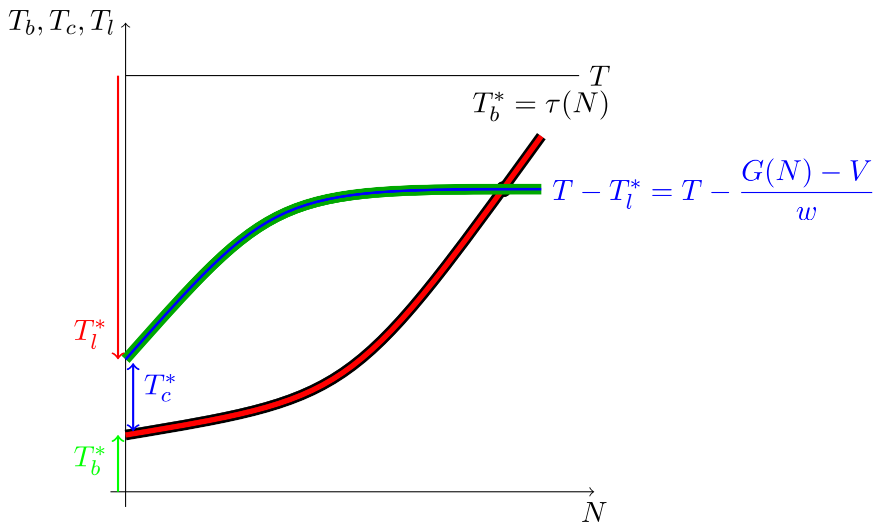


Figure 3.11: Time allocations with the number of market options for ordinary consumers

In figure 3.11, the shopping time floor constraint is depicted by the red

[3]Proposition 3.5.12 clarifies this issue by explaining optimal free time ($T_c^*$) pattern with respect to changes in the number of options ($N$).

curve, while the curve for $T_b^*$ is represented by the black curve; both coincide since the constraint binds. Similarly, the budget constraint is represented by the green curve, while the time $T_l^*$ is represented by the distance from the top horizontal line which represents $T$ to the blue curve; again, in this case both coincide since this constraint also binds. The optimal time paths can only lie between the green and the red curves. Time allocations for each number of market options are represented in figure 3.11 as follows: the light green arrow –between the $x$-axis and the black curve– indicates the shopping time, the light blue arrow –between the black and blue curves– represents the free time, and the light red arrow –between the top horizontal total time $T$-line and the blue curve– shows the working time.

Changes in the value function can be obtained through the envelope theorem of Lagrangian optimization, as follows:

$$\frac{\partial W}{\partial N} = \left.\frac{\partial L}{\partial N}\right|_* = -\mu^* \tau'(N) - \lambda^* G'(N). \tag{3.33}$$

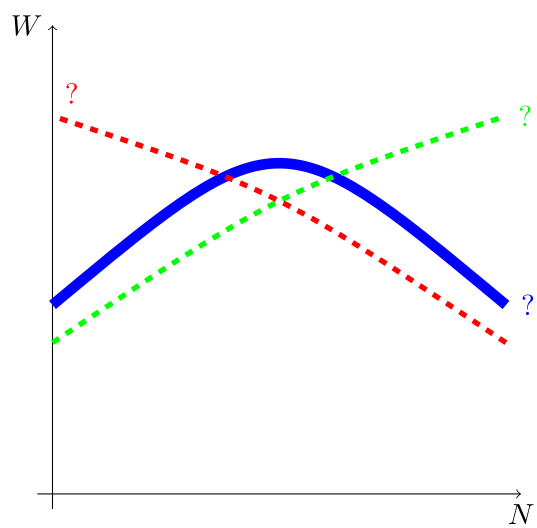


Figure 3.12: Undetermined behavior of welfare ($W$) as the number of market options ($N$) increases for the ordinary consumer.

Figure 3.11 suggests two generic forms for the functions $G(N)$ and $\tau(N)$, in which the first is strictly decreasing and the second strictly increasing.

These two functions determine the shape of the green and red curves which delimit the choice set for feasible time allocations in figure 3.11. However, it is not clear under which conditions this case will generate the paradox of choice, i.e. a welfare function vs. $N$ as the blue curve in figure 3.12 (or as in the extreme case of choice overload represented by the red curve in figure 3.12) or in contrast an alternative situation of monotonically increasing welfare, as represented by the green curve in figure 3.12. We will discuss this in depth in section 3.5, where a set of sufficient conditions is provided in a formal analysis.

**Remark** *Flat $G(N)$ or flat $\tau(N)$.*

Nevertheless, there are two extreme cases which are worth mentioning here: those of a flat shape for either $G(N)$ or $\tau(N)$. The first one would refer to a situation in which prices for all options in the market are the same, so investing time in searching does not have an impact in getting a better deal in the market. The second one considers a shopping behaviour in which by investing a fixed amount of time –independent from the total number of market options– yields full knowledge about all options. As a result, it follows that whenever $G(N)$ is flat and $\tau(N)$ is not so, welfare is always decreasing as the number of market options grows, –as it is shown with the red curve in figure 3.12–, since there is no incentive to check more options in the market. In the case that $\tau(N)$ is constant and $G(N)$ is decreasing, welfare is always increasing, –as it can be observed in the green curve in figure 3.12–, since the cost in terms of time of checking more market options does not depend on the number of options.

It is probably a common belief in modern societies that expression (3.33) should always be positive, that is, *the more, the better*. However, in the expression (3.33) there is a balance between the positive increase in welfare due to the individual assessment of savings when searching and checking more market options, on the one hand, and a negative response in welfare derived from how the individual assesses the entailed cost of time of searching. This balance may be negative at some point, that is, from some value of $N$ on. This is the situation of the paradox of choice: whereas initially the more options, the better, after some threshold number of options welfare decreases. What are the conditions under which this situation takes place? We will give an answer to this interesting question in section 3.5.

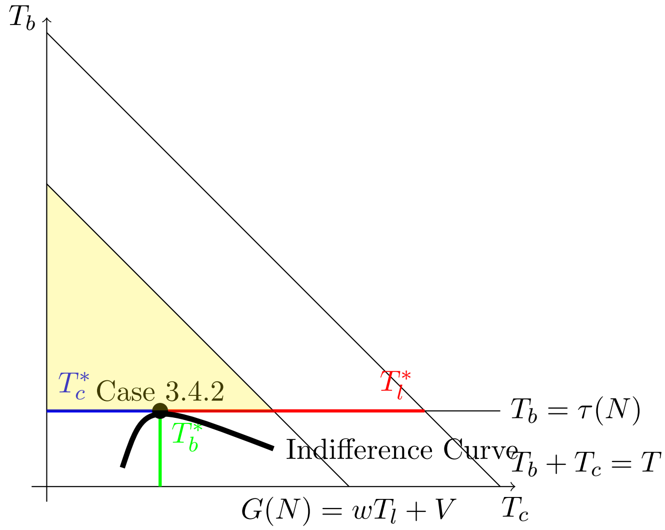


Figure 3.13: Case of workaholics, described in 3.4.2

### 3.4.2 Case of workaholics

This is the kind of solutions represented in figure 3.13. This type of consumers prefer to work more hours than enjoy more free time, and their shopping time is the least possible. In this case the budget constraint is not binding –they work too much– while the shopping time floor is always reached. Optimal solutions must satisfy the following conditions[4]:

$$\frac{\partial U}{\partial Z_2}\frac{\partial Z_2}{\partial T_c} = \frac{\partial U}{\partial Z_3}\frac{\partial Z_3}{\partial T_l}, \tag{3.34}$$

[4]Notice that in particular we have from (3.34) and (3.12)-(3.13) that $\mu^*$ is also given by

$$\mu^* = \frac{\partial U}{\partial Z_3}\frac{\partial Z_3}{\partial T_l} - \frac{\partial U}{\partial Z_1}\frac{\partial Z_1}{\partial T_b}.$$

So it must hold that

$$\left.\frac{\partial U}{\partial Z_3}\frac{\partial Z_3}{\partial T_l}\right|_* \geq \left.\frac{\partial U}{\partial Z_1}\frac{\partial Z_1}{\partial T_b}\right|_*,$$

in this case.

$$T_b^* = \tau(N),$$

$$T_c^* = T - T_l^* - \tau(N),$$

$$T_l^* = T - T_b^* - T_c^* > \frac{G(N) - V}{w},$$

$$\lambda^* = 0,$$

$$\mu^* = \frac{\partial U}{\partial Z_2}\frac{\partial Z_2}{\partial T_c} - \frac{\partial U}{\partial Z_1}\frac{\partial Z_1}{\partial T_b} \geq 0,$$

$$\delta^* = 0 = \varepsilon_c^*,$$

where all derivatives are evaluated at the optimum.

As for the ordinary consumers, conditions $\dfrac{\partial Z_1}{\partial T_b} < 0$ and $\dfrac{\partial Z_2}{\partial T_c} > 0$ guarantee that $\mu > 0$ and thus $T_b = \tau(N)$ from (3.16), so that workaholics spend the least possible time shopping. Notice, however, that those conditions are not necessary and this case can be produced under more general assumptions.

Changes in wage rate, savings and total time ($w$, $V$ and $T$) have an impact on time use patterns and the subjective shadow values of time ($\mu$), but not in the shadow value of money ($\lambda$). Any monetary effect has no impact for the workaholic: neither time use patterns nor the shadow value of money are affected by changes in wage rate or savings. An increase in total time $T$ would produce an increase in both working and free time.

The following expressions show how welfare is affected when the parameters of the problem change. They can be obtained from the envelope theorem and (3.11):

$$\frac{\partial W}{\partial w} = \left.\frac{\partial L}{\partial w}\right|_* = 0,$$

$$\frac{\partial W}{\partial V} = 0.$$

A wage raise or an increase in savings for a workaholic thus do not affect optimal welfare since her shadow value of money is zero.

**Choice overload behaviour for workaholics**

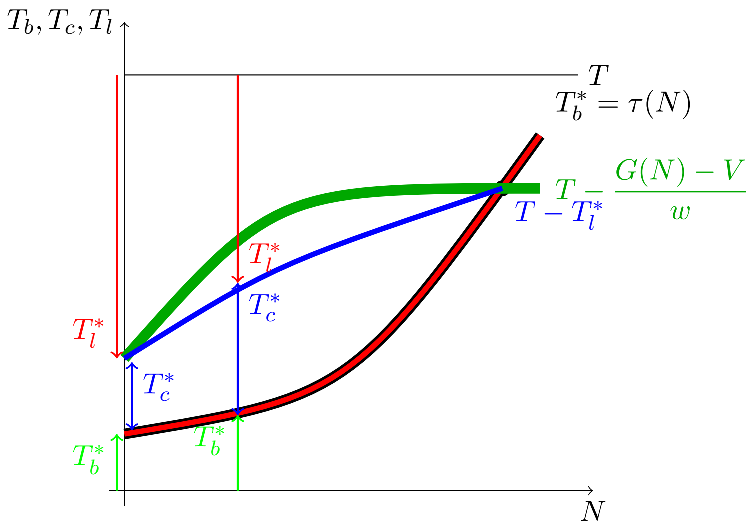


Figure 3.14: Time allocations with the number of market options for workaholics

Figure 3.14 shows feasible time use patterns between the green and red curves. Possible optimal time use paths are represented by black and blue curves. For any number of market options time allocations are represented with the coloured arrows: the light green arrow –between the $x$-axis and the black curve– indicates shopping time, the light blue arrow –between the black and blue curves– represents free time, and the light red arrow –between the top horizontal $T$-line and the blue curve– shows working time.

An increase in the number of market options ($N$) has an impact on time use patterns and the subjective shadow values of time ($\mu$), but not in the shadow value of money ($\lambda$). Such increase implies an increase in shopping time, which is obviously linked to a fall in the sum of free time and working time; the distribution between working time and free time of this reduction is carried out according to the preferences of the individual. The impact on the shadow value of shopping time $\mu$ is uncertain.

The value function is affected if the number of market options in the choice set varies. According to the envelope theorem and (3.11), this impact is defined by the expression

$$\frac{\partial W}{\partial N} = -\mu^* \frac{\partial \tau(N)}{\partial N} < 0. \tag{3.35}$$

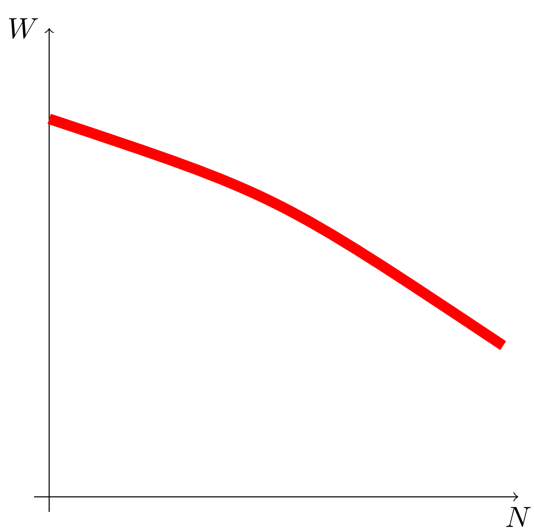


Figure 3.15: Welfare ($W$) as the number of market options ($N$) increases for the workaholic.

This result may provoke a stimulating discussion. Our western culture may lead us to be extremely focused on our work, which is very time consuming. Concerning changes in the number of market options, what do we do when we have to buy something in these circumstances? We probably buy the first thing we find. That may sound familiar. Workaholics take the first option, since they cannot waste time shopping. This follows from (3.35) and is depicted in figure 3.15. This implies that workaholics are better off if the market does not offer them more options. They do not want many choices, since it makes the act of choosing an unpleasant task which increases with the numbers of options.

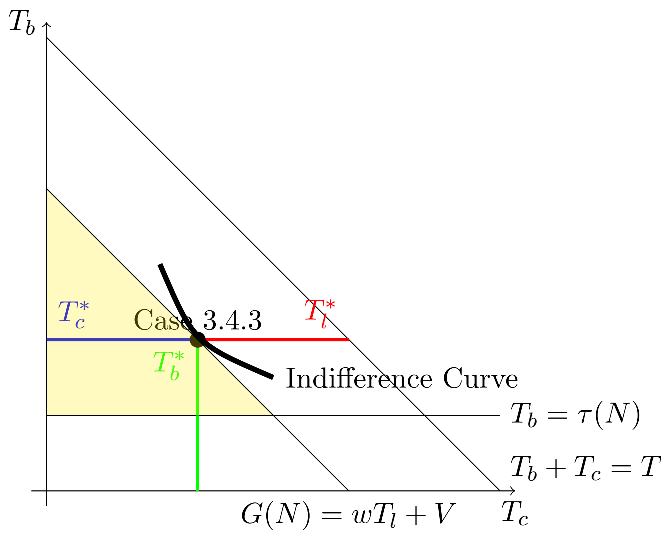


Figure 3.16: Case of shopping lovers, described in 3.4.3

### 3.4.3 Case of shopping lovers

Our consumer culture produces a lot of people who do actually enjoy shopping. A shopping lover spends all money to meet her budget. She thus spends more time shopping than the minimum imposed by shopping time floor $\tau(N)$. Optimal solutions satisfy the following conditions in this case:

$$\frac{\partial U}{\partial Z_1}\frac{\partial Z_1}{\partial T_b} = \frac{\partial U}{\partial Z_2}\frac{\partial Z_2}{\partial T_c},$$

$$T_b^* > \tau(N),$$

$$T_c^* = T - T_l^* - T_b^*,$$

$$T_l^* = \frac{G(N) - V}{w},$$

$$\lambda^* = \frac{1}{w}\left(\frac{\partial U}{\partial Z_2}\frac{\partial Z_2}{\partial T_c} - \frac{\partial U}{\partial Z_3}\frac{\partial Z_3}{\partial T_l}\right) \geq 0,$$

$$\mu^* = 0,$$

$$\delta^* = 0 = \varepsilon_c^*,$$

where all derivatives are evaluated at the optimum.

A wage raise would imply a decrease in working time that will be balanced by an increase in free time; neither shopping time nor its shadow value ($\mu$) will be affected. Similarly, an increase in savings would have the same impacts than a wage raise.

From the envelope theorem and (3.11), the value function of the problem for shopping lovers is affected by changes in the parameters as follows:

$$\frac{\partial W}{\partial w} = \left.\frac{\partial L}{\partial w}\right|_* = \lambda^* \frac{G(N) - V}{w} \geq 0,$$

$$\frac{\partial W}{\partial V} = \left.\frac{\partial L}{\partial V}\right|_* = \lambda^* \geq 0,$$

$$\frac{\partial W}{\partial T} = \left.\frac{\partial L}{\partial T}\right|_{*} = \frac{\partial U}{\partial Z_3}\frac{\partial Z_3}{\partial T_l} + \lambda^* w > 0.$$

We conclude from these expressions that either a wage raise or an increase in savings cannot decrease optimal welfare of a shopping lover since her shadow value of money is non-negative. Also, if a shopping lover could have more time (an increase in $T$, i.e. the 25-th hour of a day) she would be better off since she would allocate that extra time to shopping and to free time according to her preferences. Thus, any change that increases her income would typically improve her welfare, and so it does an increase in total time; that increase in total time would be spent in shopping time and free time, but not in working time.

**Choice overload behaviour for shopping lovers**

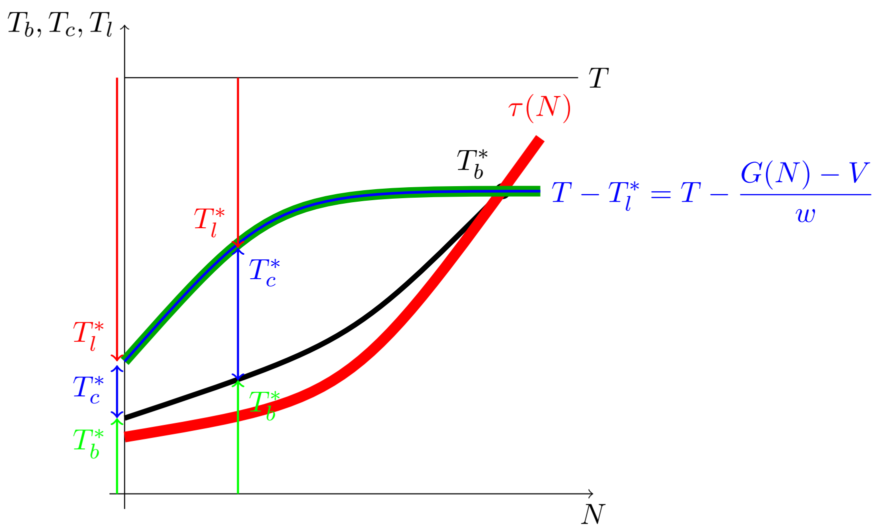


Figure 3.17: Time allocations with the number of market options for shopping lovers

Feasible time use patterns are represented in figure 3.17 between the green and red curves. Optimal time use paths are described by black and blue curves. Time is allocated for any number of market options by the coloured arrows: the light green arrow –between the $x$-axis and the black curve– indicates the shopping time, the light blue arrow –between the black and blue curves– represents the free time, and the light red arrow –between the top horizontal $T$-line and the blue curve– shows the working time.

An increase in the number of market options reduces working time; therefore, the released amount of time is allocated between shopping time and free time according to the preferences, which in general should imply an increase in both of them.

Similarly, when the number of market options vary the effect on welfare of a shopping lover is given by

$$\frac{\partial W}{\partial N} = -\lambda^* G'(N) \geq 0.$$

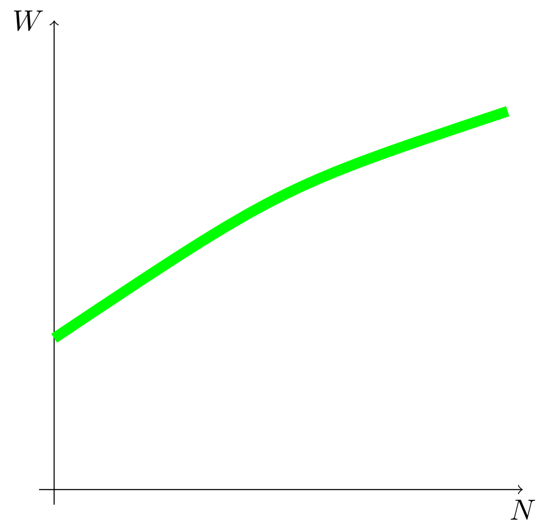


Figure 3.18: Welfare ($W$) as the number of market options ($N$) increases for a shopping lover.

In the typical case that $G'(N) < 0$ and $\lambda > 0$, welfare increases as the number of market options ($N$) increases, which makes sense for shopping lovers. It follows that shopping lovers are better off if the market offers them more number of options.

### 3.4.4 Case of unconstrained consumers

This is probably the standard case for individuals provided that the number of options $N$ is not too large. Here neither budget nor other time restriction is constraining optimal decisions on the allocation of time. In this case no constraint is binding since consumers do like shopping, free and working time in a balanced manner. Optimal solutions satisfy the following conditions:

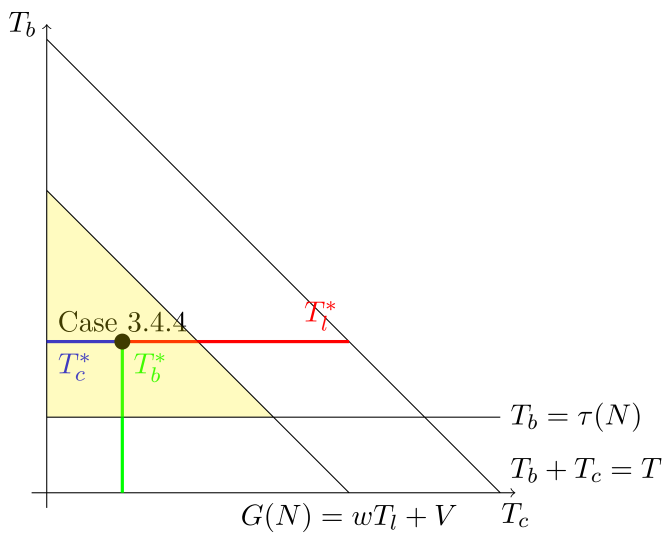


Figure 3.19: Case of unconstrained consumers, described in 3.4.4

$$\frac{\partial U}{\partial Z_1}\frac{\partial Z_1}{\partial T_b} = \frac{\partial U}{\partial Z_2}\frac{\partial Z_2}{\partial T_c} = \frac{\partial U}{\partial Z_3}\frac{\partial Z_3}{\partial T_l}, \tag{3.36}$$

$$T_b^* > \tau(N),$$

$$T_c^* = T - T_l^* - T_b^*,$$

$$T_l^* > \frac{G(N) - V}{w},$$

$$\lambda^* = \mu^* = \delta^* = \varepsilon_c^* = 0,$$

where all derivatives are evaluated at the optimum.

Time use patterns and shadow values of shopping time and money are not affected by slight changes in the parameters, as expected for an unconstrained optimum. Similarly, the value function of the problem in this case is not affected by changes in the parameters $w$ or $V$.

Welfare is not affected by changes in wages and savings. However, if an unconstrained consumer could have more time (an increase in $T$, i.e. the 25-th hour of a day) she would be better off since she would allocate that extra time to shopping time, to free time and to working time according to her preferences. Notice that an unconstrained solution requires that

$$\frac{\partial Z_1}{\partial T_b} > 0, \frac{\partial Z_2}{\partial T_c} > 0, \frac{\partial Z_3}{\partial T_l} > 0,$$

because of (3.7) and (3.36). It follows from the envelope theorem of unconstrained optimization that

$$\frac{\partial W}{\partial T} = \left.\frac{\partial U}{\partial Z_3}\frac{\partial Z_3}{\partial T}\right|_* > 0,$$

so that welfare increases when $T$ rises slightly, as claimed above.

Therefore, neither more savings nor a wage raise affect to her welfare; welfare would only be positively affected by having more time.

#### Choice overload behaviour for unconstrained consumers with flat $\tau(N)$

It is shown in figure 3.20, again, that feasible time use patterns can only lie between the green and red curves. Optimal time use paths are described by

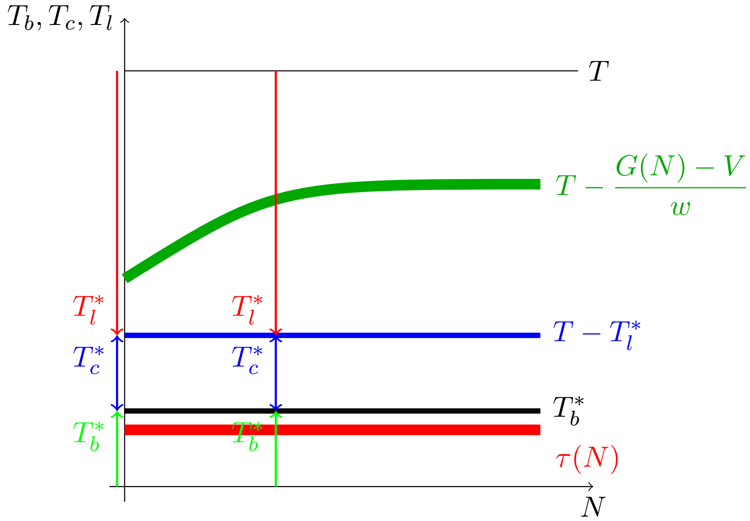


Figure 3.20: Time allocations with the number of market options for unconstrained consumers with a flat $\tau(N)$.

black and blue curves. Time is allocated for any number of market options by the amounts represented with the coloured arrows: the light green arrow —between the $x$-axis and the black line– indicates the shopping time, the light blue arrow –between the black and blue curves– represents the free time, and the light red arrow —between the top horizontal $T$-line and the blue curve– shows the working time.

In the ideal case of a flat shopping time floor ($\tau'(N) = 0$) as shown in figure 3.20, together with a utility structure such that the $\tau(N)$-constraint does not hold, welfare is invariant as the number of options increases. This is because the optimal choice of the consumer remains feasible for any number of options $N$ in figure 3.20.

Under the ideal condition of $\tau(N)$ flat, the unconstrained consumer does not improve with the number of options in the market, that is

$$\frac{\partial W}{\partial N} = 0.$$

However, the following possibility may be considered typical for an un-

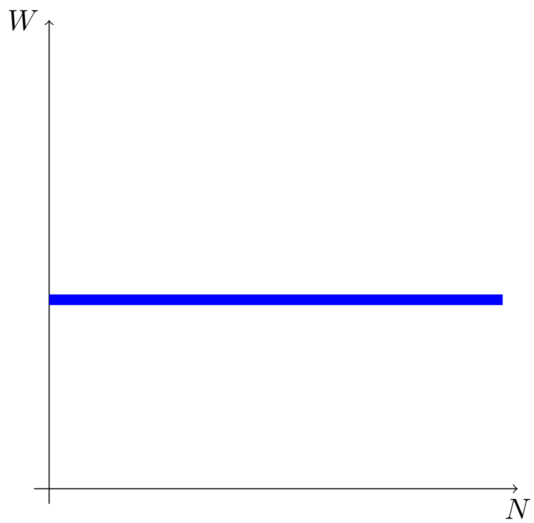


Figure 3.21: Welfare ($W$) as the number of market options ($N$) increases for the unconstrained consumer with a flat $\tau(N)$.

constrained consumer facing an increasing $\tau(N)$: at a given value $N$ of market options which defines a choice set, an unconstrained consumer can implement her favourite time allocation. Nevertheless, if $N$ increases dramatically, the favourite allocation is no longer feasible and the unconstrained consumer might become a workaholic, since its optimal solution in the time triangle is swept by the upwards shift of $\tau(N)$. Such conversion will ultimately have an impact on welfare, which starts to decrease as we already discussed in the case of workaholics. This forced conversion of an unconstrained consumer into a workaholic when $N$ increases enough seems common and realistic. A consumer who is unaffected by a low number of options eventually enters into choice overload and his welfare is negatively affected. This could be considered the case of *typical consumers*.

## 3.5 Accounting for choice overload situations

A detailed discussion of the main possible solutions of the model have been offered in the previous section. As discussed there, in the case of ordinary consumers, some opposite effects left the door open to the paradox of choice (see (3.12)). Within this section we analyse more in depth how choice overload may appear in the model and what are the conditions which generate both the paralysis effect (subsection 3.5.1) and the paradox of choice (subsection 3.5.2).

We must stress on the subtle differences and strong relations between choice overload and the paradox of choice. In its initial meaning (Iyengar and Lepper, 2000), choice overload makes reference to smaller choice sets being more appealing for the individual than larger choice sets; in other words, the motivation of an individual when facing a choice problem which contains 10 options is greater than when facing 200 options. Notice that in principle here nothing is said about welfare of the consumer when choosing. It is currently understood that choice overload is a term that refer to any (negative) issue affecting the consumer when the number of options is large enough. The paradox of choice refers to a decrease in welfare when a number of options in the choice set is reached, which makes the act of choosing distressing and eventually overwhelming. This feature is summarized in an inverted U-shape in a graph showing welfare versus the number of options. A phenomenon which puts together both the choice overload and the paradox of choice can be said to be the so called paralysis effect (Schwartz, 2005); that refers to an extreme situation in which the choice overload is such that completely demotivates the act of choosing.

The case of ordinary consumers may account for the paradox of choice and the paralysis. However, these situations might not occur always. Within this section we provide conditions for both phenomena.

Let us start by considering the following assumptions over the shopping time floor $\tau(N)$.

Given a $C^2$ function $\tau : \Re_+ \to \Re_+$ consider the following properties:

**Assumption** [ $\tau$**1** ]

$$\tau(0) = \tau_0 < T. \tag{3.37}$$

**Assumption** [ $\tau$**2** ]

$$\tau'(N) \geq 0 \; for \; N \geq 0. \tag{3.38}$$

**Assumption** [ $\tau$**3** ]

$$\tau(N) \longrightarrow \infty \text{ as } N \longrightarrow \infty. \tag{3.39}$$

**Assumption** [ $\tau$**4** ]

$$\tau''(N) \geq 0 \; for \; N \geq 0. \tag{3.40}$$

Notice that (3.38) –with strict inequality– plus (3.40) imply (3.39), but they may be considered independently below.

Let us also consider the following assumptions over the expenditure function $G(N)$.

Given a $C^2$ function $G : \Re_+ \to \Re_+$ consider the following properties:

**Assumption** [ $G$**1** ] For some constant $\overline{G}$, we have

$$G(N) > \overline{G} \; for \; N \geq 0. \tag{3.41}$$

**Assumption** [ $G$**2** ]

$$G'(N) \leq 0 \; for \; N \geq 0. \tag{3.42}$$

**Assumption** [ $G$**3** ]

$$G(N) \longrightarrow \overline{G} \geq V \text{ as } N \longrightarrow \infty. \tag{3.43}$$

**Assumption** [ $G$**4** ]

$$G''(N) > 0 \; for \; N \geq 0. \tag{3.44}$$

A non increasing function bounded from below must converge. Property (3.43) above thus follows from (3.41) and (3.42). Also, (3.43) implies

**Assumption** [ $G$**5** ]

$$G'(N) \longrightarrow 0 \text{ as } N \longrightarrow \infty.$$

The proof is included here for the sake of completeness.

**Lemma 3.5.1** *[ G3 ] ⇒ [ G5 ]*

**Proof** Assume that $G'(N) \longrightarrow H \neq 0$ as $N \longrightarrow \infty$ and fix $h$ such that $0 < h < |H|$. Let $N_1$ be such that $|G'(N)| > h$ for all $N > N_1$. Let $\varepsilon > 0$. [ $G3$ ] implies that there is some $N_2$ such that $|G(N) - \overline{G}| < \varepsilon$ for all $N > N_2$. Now take any pair $N, N' > \max\{N_1, N_2\}$ such that $|N - N'| > \frac{2\varepsilon}{h}$. Since $N, N' > N_2$ it follows that

$$|G(N) - G(N')| \leq |G(N) - \overline{G}| + |G(N') - \overline{G}| < 2\varepsilon.$$

On the other hand, since $N, N' > N_1$, the mean value theorem implies that

$$|G(N) - G(N')| \geq \min_{n \in [N, N']} |G'(n)||N - N'| > h|N - N'| > 2\varepsilon,$$

which is a contradiction. It thus follows that $H = 0$. **Q.E.D.**

Let us describe some preliminary formal definitions which will be used later to prove our results.

**Definition 3.5.2** *(Choice structure)*

*A **choice structure** is a quadruple*

$$(\tau, G, V, w)\,, \tag{3.45}$$

*where* $\tau, G$ *are defined as above,* $0 < V \leq G$ *and* $w > 0$.

**Definition 3.5.3** *(N-choice set)*

*Given a choice structure, for each $N \geq 0$ the $N$-**choice set** is defined by*

$$\Omega(N) = \left\{ \vec{T} \in \Delta : T_b \geq \tau(N), G(N) \leq wT_l + V \right\},$$

*where $\Delta$ is the time simplex defined in (3.3).*

**Definition 3.5.4** *(N-feasible time allocation)*

*For $N \geq 0$, a time allocation $\vec{T} \in \Omega(N)$ is called **N-feasible**.*

**Definition 3.5.5** *($\gamma$ function)*

*The $\gamma(N)$ function is defined by*

$$\gamma(N) = T - \frac{G(N) - V}{w}. \tag{3.46}$$

Since the budget constraint $\frac{G(N)-V}{w} \leq T_l$ imposes a time floor for $T_l$, the $\gamma$-curve defines a time ceiling for the non-working time, i.e. $T_b + T_c \leq \gamma(N)$ for each $N$.

**Definition 3.5.6** *(N-proper choice structure)*

*For $N \geq 0$, the choice structure is $N$-**proper** if $\gamma(N) > \tau(N)$.*

Since time allocations $\vec{T}$ with $0 \leq T_c \leq \gamma(N) - \tau(N)$ are $N$-feasible and $\gamma(N) - \tau(N) > 0$ if the choice structure is $N$-proper, $\Omega(N)$ is a continuum (in particular, $\text{card}\Omega(N) > 1$) and the $N$-choice set is non-trivial in this case.

**Definition 3.5.7**

- *If* $\gamma(0) \geq \tau(0)$ *we say that the choice structure starts at* $N_0 = 0$*.*
- *If* $\gamma(0) < \tau(0)$*, let* $N_0$ *be the largest value such that the choice structure is not* $N$*-proper for* $0 \leq N \leq N_0$*. If* $N_0 < +\infty$ *we say that the choice structure starts at* $N_0$*.*

  *Notice that* $N_0 > 0$ *is the minimum value at which the curves* $\gamma(N)$ *and* $\tau(N)$ *intersect transversally.*

A choice structure which is improper (that is, $N_0 = +\infty$ above) never starts, and it is uninteresting for our purposes.

**Definition 3.5.8** *A shopping time floor* $C^2$ *function* $\tau$ *satisfying assumptions [*$\tau$*1], [*$\tau$*2] and [*$\tau$*3] is called basic. Similarly, an expenditure* $C^2$ *function* $G$ *satisfying assumptions [G1], [G2] and [G3] is called basic.*

**Definition 3.5.9** *(Basic choice structure)*

*A choice structure* $(\tau, G, V, w)$*, with* $\tau, G$ *basic, starting at some* $0 \leq N_0 < +\infty$ *and such that*

[ $\tau G$ ] *For all* $N \geq N_0$ *we have either*

$$\tau'(N) \neq 0 \text{ or } G'(N) \neq 0, \tag{3.47}$$

*is called a* ***basic choice structure****.*

A typical choice structure starting at $N_0$ can be represented as shown in figure 3.22.

In the case that $N_0 > 0$, price deals $G(N)$ for $0 \leq N \leq N_0$ are not competitive enough and also the fixed cost is so large that the sum of the required search time and working time exceeds the total time $T$ and then no feasible time distribution is possible. As the number of product options increases a better deal can be obtained, what entails a decrease in working time. If this decrease is faster than the increase in shopping time -as it occurs in the case of the figure 3.22 -, feasible time distributions emerge and the choice problem makes sense. Figure 3.23 shows a typical choice structure that starts at $N_0 = 0$. A choice structure that never starts is represented in figure 3.24.

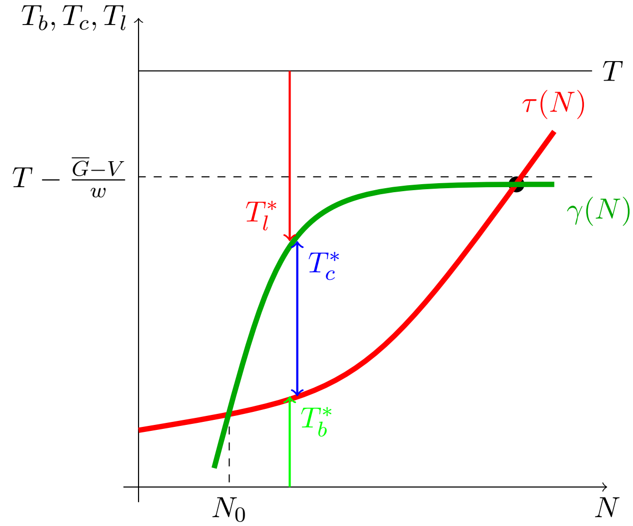


Figure 3.22: A typical choice structure starting at $N_0 > 0$

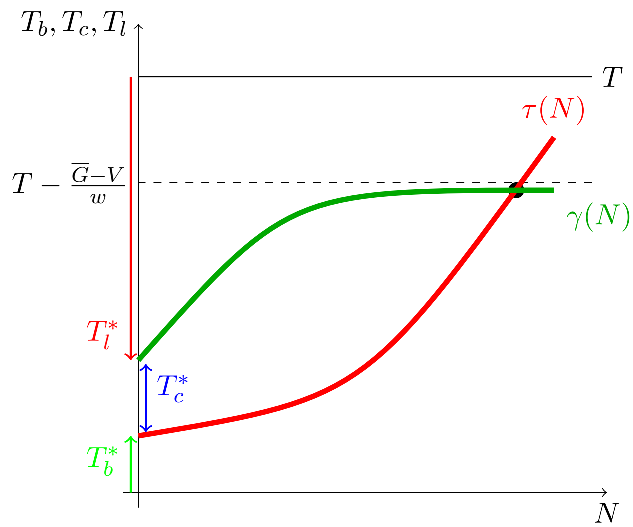


Figure 3.23: A typical choice structure starting at $N_0 = 0$

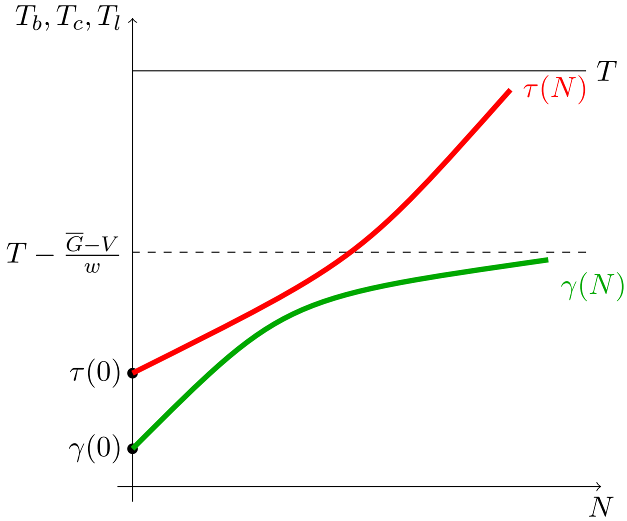


Figure 3.24: A choice structure that never starts

### 3.5.1 Choice overload and paralysis effect

Our discussion in section 3.4 suggests that a consumer (not only an ordinary consumer, but also workaholics, shopping lovers and even unconstrained consumers) might not check all the available options in the market when searching for the best deal in the market. However, so far we have not formally analysed such circumstance. We give below sufficient conditions for a basic structure to produce choice paralysis, an extreme effect of choice overload. We make use of the previous definitions, in particular of the auxiliary function $\gamma(N)$ which plays a key role.

Notice that the $\gamma(N)$ ceiling is represented by the green curve in all figures in the previous section (figures 3.11, 3.14, 3.17 and 3.20) and in all figures in this section (figures 3.22, 3.23 and 3.24).

**Proposition 3.5.10** *(Choice overload: the paralysis effect)*

*Given a basic choice structure that starts at $N_0 \geq 0$, there exists a unique $\overline{N} \geq N_0$ such that $\Omega(N) = \emptyset$ for all $N > \overline{N}$. Moreover, $\overline{N}$ satisfies*

$$\gamma(\overline{N}) = \tau(\overline{N})$$

*and* $(\gamma(N) - \tau(N))\left(N - \overline{N}\right) < 0$ *for all* $N > N_0, N \neq \overline{N}$.

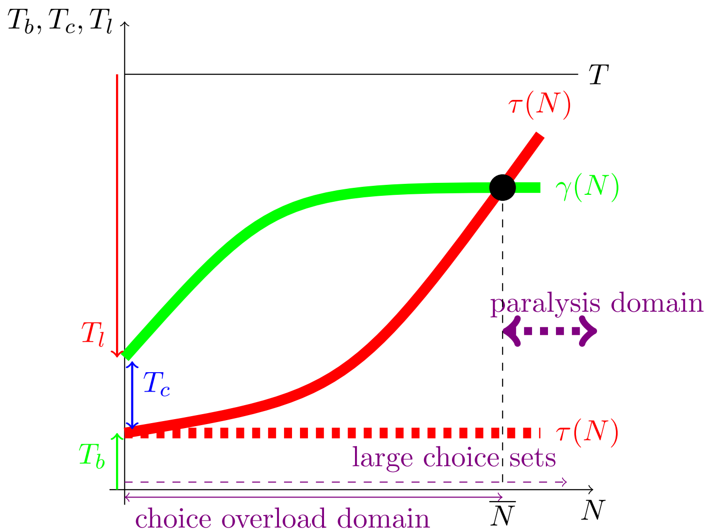


Figure 3.25: Choice overload: the paralysis effect

Notice that the emptiness of the $N$-choice set $\Omega(N)$ is the mathematical counterpart of the psychological phenomenon known as paralysis effect (Schwartz, 2000, 2005) which leads the consumer to abandon the choice problem.

**Proof** Consider $f(N) = \gamma(N) - \tau(N)$, where $\gamma(N)$ is the function defined in (3.46). It follows that that $f(N) > 0$ for some $N > N_0$ because the choice structure starts at $N_0$. Also, $[\tau 3]$ and $[G3]$ imply that $f(N) \longrightarrow -\infty$ as $N \longrightarrow \infty$. The existence of $\overline{N}$ such that $f(\overline{N}) = 0$ follows from the continuity of $f$. Since $G'(N)$ and $\tau'(N)$ do not vanish simultaneously (see

$[\tau G]$ in definition 3.5.9), we have that $f'(N) = -\frac{G'(N)}{w} - \tau'(N) < 0$ from $[\tau 2]$ and $[G2]$. Notice that $\Omega(N) = \left\{\vec{T} \in \Delta : \tau(N) \leq T_b + T_c \leq \gamma(N)\right\}$ so that $\Omega(N) = \emptyset$ for $N > \overline{N}$. It also follows that $\overline{N}$ is unique and, therefore $(\gamma(N) - \tau(N))\left(N - \overline{N}\right) < 0$ for all $N > N_0, N \neq \overline{N}$. **Q.E.D.**

**Remark** There are two interesting extreme cases that are not included above:

1. (Flat $G$) $\tau(N)$ basic with $\tau'(N) > 0$ for all $N \geq 0$ and $G(N) = \overline{G}$ =constant with $\tau(0) < T - \frac{\overline{G}-V}{w}$. The argument in the proof applies and the conclusion remains valid in this case.

2. (Flat $\tau$) $G(N)$ basic with $G'(N) < 0$ and $\tau(N) = \tau_0$ =constant for all $N \geq 0$ with $\tau_0 < T - \frac{\overline{G}-V}{w}$, where $\overline{G}$ is as in $[G3]$ (see (3.43)). In this case, $\overline{N} = +\infty$ and the choice problem never terminates.

Figure 3.25 illustrates how beyond some number $\overline{N} \geq 0$ of market options there is no feasible allocation of time uses for the consumer, which can be interpreted as if the choice task is so overwhelming that the choice problem is eventually abandoned. Thus, a consumer facing a choice set with size beyond $\overline{N}$ experiences the so called paralysis effect: the size of the choice set is so big that it lies on what we have termed in figure 3.25 as the *paralysis domain*.

### 3.5.2 Choice overload and the paradox of choice

Consumers may start to feel dissatisfied when they confront a feasible choice set with a high number of checked options. This is precisely the content of the paradox of choice, which means that you are *"doing better, but feeling worse"* (Schwartz, 2005). This intriguing phenomenon defies a paradigm within consumption based societies. As we introduced in previous section, there is room for this in the case of ordinary consumers. We provide below sufficient conditions for the paradox of choice to happen. Let us formulate conditions that produce as solutions of the model the case of ordinary consumers and analyse its behaviour with respect to $N$.

We consider the utility structure composed by assumption $[U1]$ in (3.7) and the following:

**Assumption** [ $Z1$ ]

$$\frac{\partial U}{\partial Z_2}\frac{\partial Z_2}{\partial T_c} > \max\left\{\frac{\partial U}{\partial Z_1}\frac{\partial Z_1}{\partial T_b}, \frac{\partial U}{\partial Z_3}\frac{\partial Z_3}{\partial T_l}\right\}, \tag{3.48}$$

for all $\vec{T} \in \Delta$, where intermediate derivatives are evaluated at $\vec{Z}(\vec{T}) = (Z_1(T_b), Z_2(T_c), Z_3(T_l))$.

Notice that assumption [$Z1$] in (3.48) is a general assumption that is satisfied in the following cases:

- $\frac{\partial Z_2}{\partial T_c} > 0$, $\frac{\partial Z_1}{\partial T_b} > 0$ and $\frac{\partial Z_3}{\partial T_l} < 0$, and also $\frac{\partial U}{\partial Z_2}\frac{\partial Z_2}{\partial T_c} > \frac{\partial U}{\partial Z_1}\frac{\partial Z_1}{\partial T_b}$.
- $\frac{\partial Z_2}{\partial T_c} > 0$, $\frac{\partial Z_1}{\partial T_b} < 0$ and $\frac{\partial Z_3}{\partial T_l} < 0$.

Given a basic choice structure and the utility structure (that is, some utility function $U$ and wants mappings $Z_m$), the $N$-choice problem consists of finding the $N$-feasible time allocation, that is $\vec{T} \in \Omega(N)$, that maximizes $U(Z_1(T_b), Z_2(T_c), Z_3(T_l))$. It follows from Weiertrass' theorem that an optimal allocation

$$\vec{T}^* \equiv \vec{T}^*(N) = \arg\max\left\{U(Z_1(T_b), Z_2(T_c), Z_3(T_l)) : \vec{T} \in \Omega(N)\right\}$$

exists for every $N$ such that $\Omega(N)$ is non-empty.

**Proposition 3.5.11** *(Sufficient conditions for ordinary consumers)*

*Consider a basic choice structure that starts at $N_0 \geq 0$. Assume that $U$ and $Z_m$ satisfy assumptions [U] and [Z1] above. Let $\overline{N}$ be as in proposition 3.5.10. Then, for $N_0 \leq N \leq \overline{N}$, the optimal allocation of time $\vec{T}^* = \vec{T}^*(N)$ is given by expressions $T_b^* = \tau(N)$, $T_c^* = T - \tau(N) - \frac{G(N) - V}{w}$, and $T_l^* = T - T_b^* - T_c^* = \frac{G(N) - V}{w}$. Also, for $N_0 \leq N \leq \overline{N}$, it holds that $0 < T_b^*, T_c^*, T_l^* < T$.*

**Proof** The optimization problem can be considered as a 2D problem in the variables $(T_c, T_b)$ defined on the projection of the time simplex represented in figure 3.7, which is defined by

$$\Delta_2 = \{(T_c, T_b) : T_c, T_b \geq 0, T_c + T_b \leq T\}. \tag{3.49}$$

Working time is thus obtained as the residual $T_l = T - T_c - T_b$.

From [$G1$]-[$G3$] and the constraint $G(N) \leq wT_l + V$, we have $T_l \geq \frac{G(N)-V}{w}$ or equivalently $T_c + T_b \leq T - \frac{G(N)-V}{w} = \gamma(N) < T$.

Also, [$\tau 1$], [$\tau 2$] and the time floor implies that $T_b \geq \tau(N) > 0$.

The domain of the 2D problem can thus be written as

$$\Omega_2(N) = \{(T_c, T_b) \in \Delta_2 : T_c \geq 0, T_c + T_b \leq \gamma(N), T_b \geq \tau(N)\}.$$

With this formulation, the Lagrangian of the problem is

$$\begin{aligned} L(T_b, T_c, \lambda, \mu) = \quad & U(Z_1(T_b), Z_2(T_c), Z_3(T - T_b - T_c)) \\ & -\lambda(G(N) - w(T - T_b - T_c) - V) \\ & -\mu(\tau(N) - T_b), \end{aligned}$$

where $\mu$ and $\lambda$ are the Lagrange multipliers associated with the constraints $\tau(N) - T_b \leq 0$ and $G(N) - w(T - T_b - T_c) - V \leq 0$, respectively. At the optimum, it must hold that $\mu \geq 0$ and $\lambda \geq 0$, and

$$G(N) - w(T - T_b - T_c) - V \leq 0 (= 0 \; if \; \lambda > 0), \tag{3.50}$$

$$\tau(N) - T_b \leq 0 (= 0 \; if \; \mu > 0). \tag{3.51}$$

The optimal solution $\vec{T}^*$ must also satisfy, for some $\lambda$ and $\mu$

$$\frac{\partial U}{\partial Z_1}\frac{\partial Z_1}{\partial T_b} - \frac{\partial U}{\partial Z_3}\frac{\partial Z_3}{\partial T_l} - \lambda w + \mu = 0, \tag{3.52}$$

$$\frac{\partial U}{\partial Z_2}\frac{\partial Z_2}{\partial T_c} - \frac{\partial U}{\partial Z_3}\frac{\partial Z_3}{\partial T_l} - \lambda w \leq 0 (= 0 \; if \; T_c > 0). \tag{3.53}$$

From (3.53) and [Z1],

$$\lambda^* \geq \frac{1}{w}\left(\frac{\partial U}{\partial Z_2}\frac{\partial Z_2}{\partial T_c} - \frac{\partial U}{\partial Z_3}\frac{\partial Z_3}{\partial T_l}\right) > 0, \tag{3.54}$$

and then (3.50) implies

$$G(N) - w(T - T_b - T_c) - V = 0. \tag{3.55}$$

Also, from (3.52), (3.53) and assumption $[Z1]$ in (3.48)

$$\mu^* = \frac{\partial U}{\partial Z_2}\frac{\partial Z_2}{\partial T_c} - \frac{\partial U}{\partial Z_1}\frac{\partial Z_1}{\partial T_b} > 0,$$

so that (3.51) gives

$$T_b^* = \tau(N). \tag{3.56}$$

Solving (3.55) and (3.56) it follows that the optimal allocation of time $\vec{T}^* = \vec{T}^*(N)$ is given by expressions (3.19), (3.20) and (3.21).

This is the unique feasible time allocation satisfying all Kuhn-Tucker conditions of the optimization problem for a local maximum. Weiestrass' Theorem thus implies that the optimal allocation of time $\vec{T}^* = \vec{T}^*(N)$ given by expressions (3.19), (3.20) and (3.21) is the global solution to the problem of ordinary consumers in proposition 3.5.11. Since the choice structure is basic, $T_b > 0$ from $[\tau 1]$ and $[\tau 2]$ in (3.37) and (3.38), respectively, and $T_l > 0$ from $[G1]$ in (3.41), for all $N \geq N_0$. Also, $T_c = \gamma(N) - \tau(N) > 0$ for $N_0 < N < \overline{N}$, from proposition 3.5.10. Since $T_b + T_c + T_l = T$, it follows that $0 < T_b^*, T_c^*, T_l^* < T$. **Q.E.D.**

**Proposition 3.5.12** *(Growth and concavity of $T_c^*$ for ordinary consumers)*

*Consider a basic choice structure that starts at $N_0 \geq 0$ and satisfies [$\tau 4$] and [G4] in (3.40) and (3.44), respectively. Assume that $U$ and $Z_m$ satisfy [U] and [Z1] in (3.7) and (3.48), respectively. Let $N$ be as in proposition 3.5.10. Then, $T_c^*(N)$ is concave as a function of $N$, and furthermore* *(a) If* $N_0 = 0$ *and* $-G'(0) < w\tau'(0)$ *then* $\dfrac{\partial T_c^*}{\partial N}(N) < 0$ *for all* $0 < N < \overline{N}$.

*(b) If* $N_0 \geq 0$ *and* $-G'(N_0) > w\tau'(N_0)$ *then* $T_c^*(N)$ *reaches its unique maximum at* $N_c$*, with* $N_0 < N_c < \overline{N}$*, which is given by the identity* $-G'(N_c) = w\tau'(N_c)$.

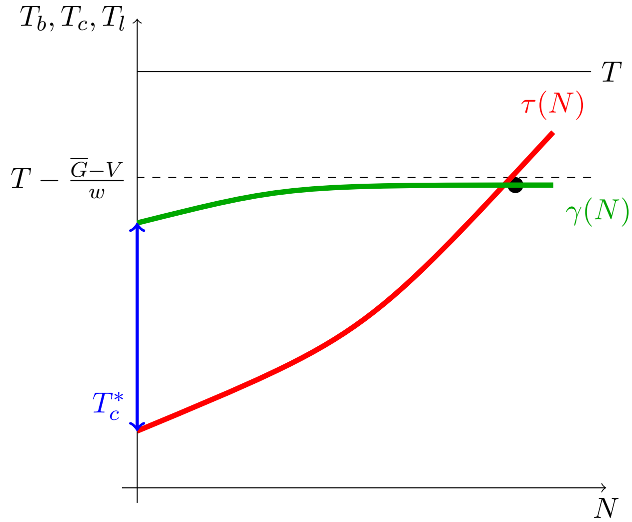


Figure 3.26: Case *(a)* in proposition 3.5.12

**Proof** Since $T_c^* = T - \tau(N) - \dfrac{G(N) - V}{w}$ , it follows from [$\tau 4$] and [$G4$] in (3.40) and (3.44), that

$$\frac{\partial^2 T_c^*}{\partial N^2}(N) = -\frac{G''(N)}{w} - \tau''(N) < 0,$$

so that $T_c^*(N)$ is concave. Notice that $\dfrac{\partial T_c^*}{\partial N}(N)$ is decreasing.

In the case that $N_0 = 0$, since $\dfrac{\partial T_c^*}{\partial N}(0) = -\dfrac{G'(0)}{w} - \tau'(0) < 0$ then $\dfrac{\partial T_c^*}{\partial N}(N) < 0$ for all $0 < N < \overline{N}$. This proves *(a)*.

If $N_0 > 0$, by definition $T_c^*(N_0) = 0$ and $T_c^*(N) > 0$ for some $N > N_0$. From continuity, $\dfrac{\partial T_c^*}{\partial N}(N) > 0$ for all $N > N_0$ sufficiently close to $N_0$. Since

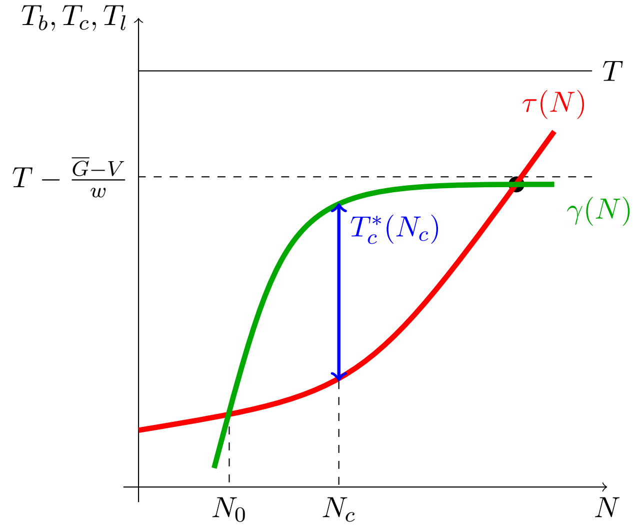


Figure 3.27: Case *(b)* in proposition 3.5.12

$T_c^*(\overline{N}) = 0$, there is some $N_c$ with $N_0 < N_c < \overline{N}$ such that $\dfrac{\partial T_c^*}{\partial N}(N) = 0$, which is unique by concavity. This proves *(b)*. **Q.E.D.**

**Remark** Given a basic choice structure with assumptions $[\tau 4]$ and $[G4]$, it is always the case that $T_b^*(N)$ is non-decreasing and convex and $T_l^*(N)$ is decreasing and convex. Now, $T_c^*$ is concave (from proposition 3.5.12) and eventually must decrease to zero (from proposition 3.5.10), but may initially increase up to a maximum in the case that the initial condition $-G'(N_0) \geq w\tau'(N_0)$ holds. This condition implies that, in terms of free time, there is incentive to start searching for the product in the market, since the gain of starting the search is greater than the entailed opportunity cost. Cases *(a)* and *(b)* considered in proposition 3.5.12 are represented in the figures (3.26) and (3.27).

**Theorem 3.5.13** *(Choice overload: paradox of choice)*

*Consider a basic choice structure that starts at* $N_0 \geq 0$ *and satisfies assumptions [*$\tau 4$*] and [G4] defined in (3.40) and (3.44), respectively. Assume that* $U$ *and* $Z_m$ *satisfy [U], [Z1] given in (3.7) and (3.48), respectively, and also*

***Assumption*** *[ Z2 ] For all* $\vec{T} \in \Delta$,

$$\frac{\partial U}{\partial Z_3}\frac{\partial Z_3}{\partial T_l}(\vec{T}) > \frac{\partial U}{\partial Z_1}\frac{\partial Z_1}{\partial T_b}(\vec{T}). \tag{3.57}$$

*Let* $N$ *and* $N_c$ *be as in proposition 3.5.12. Consider the maximal utility of the N-choice problem* $U^*(N) = \max\{U(Z_1(T_b), Z_2(T_c), Z_3(T_l))\}$.

*Then, there is* $N^* \leq N_c$ *such that* $\dfrac{\partial U^*}{\partial N}(N) < 0$ *for all* $N^* \leq N \leq \overline{N}$.

**Proof** Notice that in both cases *(a)* and *(b)* of Proposition 3.5.12 it holds that there is $N_c \geq 0$ such that

$$\frac{\partial T_c^*}{\partial N}(N) = -\frac{G'(N)}{w} - \tau'(N) < 0 \; for \; N_c < N < \overline{N}. \tag{3.58}$$

The envelope theorem gives

$$\frac{\partial U^*}{\partial N}(N) = -\lambda^*(N)G'(N) - \mu^*(N)\tau'(N).$$

This can be written as

$$\begin{aligned}\frac{\partial U^*}{\partial N}(N) = \quad & w\lambda^*(N)\left(-\frac{G'(N)}{w} - \tau'(N)\right) + \\ & + \tau'(N)\left(w\lambda^*(N) - \mu^*(N)\right).\end{aligned}$$

It follows from [$\tau 2$] and [$Z5$] in (3.38) and (3.57), respectively, and also from (3.52), that, for all $N_0 < N < \overline{N}$

$$\frac{\partial U^*}{\partial N}(N) < w\lambda^*(N)\left(-\frac{G'(N)}{w} - \tau'(N)\right) = w\lambda^*(N)\frac{\partial T_c^*}{\partial N}(N).$$

Since $\lambda^*(N) \geq 0$ for all $N_0 < N < \overline{N}$, then (3.58) implies that $\dfrac{\partial U^*}{\partial N}(N) < 0$ for all $N_c \leq N \leq \overline{N}$. This concludes the proof. **Q.E.D.**

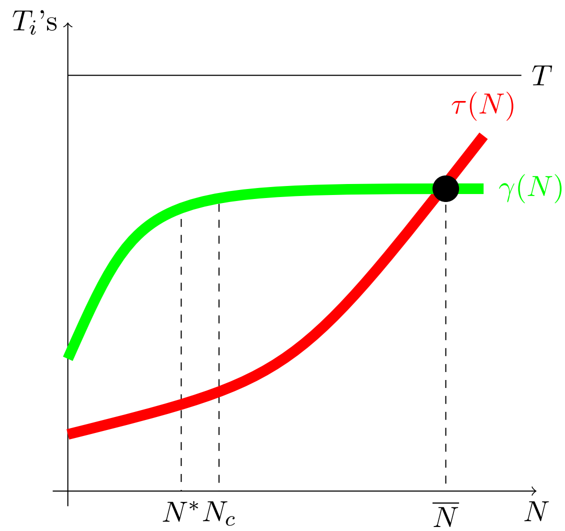


Figure 3.28: Illustration of the conditions for the existence of the paradox of choice

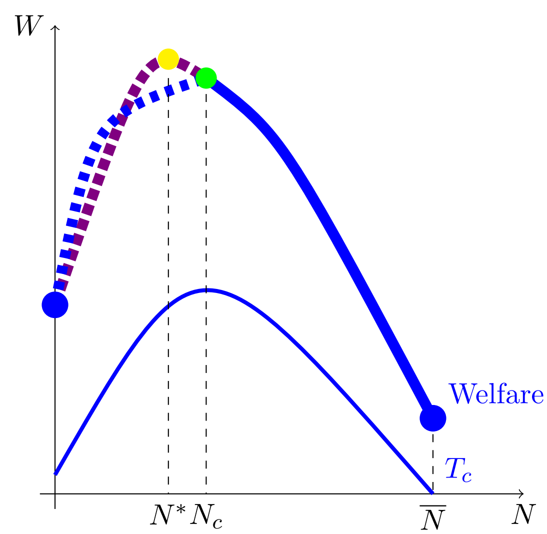


Figure 3.29: Welfare ($W$) as the number of market options ($N$) increases when conditions for the paradox of choice are fulfilled (in solid blue)

**Remark**

(1) Notice that in the case that time marginal utilities are ordered as follows

$$\frac{\partial U}{\partial Z_2}\frac{\partial Z_2}{\partial T_c}(\vec{T}) > \frac{\partial U}{\partial Z_3}\frac{\partial Z_3}{\partial T_l}(\vec{T}) > \frac{\partial U}{\partial Z_1}\frac{\partial Z_1}{\partial T_b}(\vec{T}),$$

the paradox of choice takes place, i.e. indirect utility $U^*(N)$ as a function of $N$ starts decreasing beyond certain number of product options. Under the assumptions of theorem 3.5.13 it follows that, when the number of options is such that free time starts to decline, utility is already decreasing.

(2) Under the assumptions in proposition 3.5.12 a more general utility structure also implies the paradox of choice as long as $w\lambda^*(N) < \mu^*(N)$. Since (3.52) implies

$$w\lambda^*(N) - \mu^*(N) = \frac{\partial U}{\partial Z_1}\frac{\partial Z_1}{\partial T_b}(\vec{T}) - \frac{\partial U}{\partial Z_3}\frac{\partial Z_3}{\partial T_l}(\vec{T}),$$

under $[Z2]$ in (3.57), we have $w\lambda^*(N) < \mu^*(N)$ as needed. This cannot be guaranteed in general.

(3) Notice that, under the assumptions in proposition 3.5.11, the envelope theorem implies

$$\frac{\partial U^*}{\partial N}(N) = -\lambda^*(N)G'(N) - \mu^*(N)\tau'(N).$$

The variation of utility with respect to the number $N$ of product options is thus obtained as the result of comparing the marginal costs and benefits of looking at the $(N+1)$-th version of the product. Indeed, increasing $N$ by one has two opposite effects on the size of the $N$-feasible set. On one hand, it enlarges $\Omega(N)$ by expanding the time ceiling constraint by $-\frac{G'(N)}{w}$ units, which in turn increases the value of utility by $-\lambda^*(N)G'(N)$. On the other hand, it reduces $\Omega(N)$ by contracting the time floor constraint by $\tau'(N)$ units, which decreases utility by $\mu^*(N)\tau'(N)$. That is, searching for and looking at a new unit of the product entails a gain in the deal which is valued by $-\lambda^*(N)G'(N)$ (in utils) but also a cost in time which is measured by $\mu^*(N)\tau'(N)$ (in utils). Utility keeps increasing with every extra option as long as the benefit of the gain surpasses the cost of inspection time. It turns out that, under the assumptions of theorem 3.5.13, this is never the case.

Theorem 3.5.13 provides conditions that are sufficient for the existence of a decreasing region of the welfare function, displayed with a solid blue curve in figure 3.29. Figure 3.29 shows two alternative initial patterns for the welfare function when the paradox of choice takes place under the assumptions of theorem 3.5.13. In general, we would expect the pattern represented by the dotted purple line, which reaches the maximum at the yellow point.

# Chapter 4

# Numerical analysis on time microeconomics[1]

*If your experiment needs statistics, you ought to have done a better experiment.*

(Ernest Rutherford)

This chapter is devoted to the numerical analysis of the theoretical model of chapter 3, paying attention to each of the cases considered in chapter 3, which dealt with several choice overload issues. We start with a short introduction in section 4.1, to refresh quickly previous research on choice overload. Section 4.2 is devoted to explain in detail the specific configuration of the numerical analysis of the model in chapter 3, and it is complemented with section 4.3 on data and methods. We develop two differentiated parts in the analysis: the first part is related to the main cases discussed in section 3.4, and is described in section 4.4; the second part –explained in section 4.5– is focused on a case study with actual data on market prices for a given product.

[1]The main results of the case study in section 4.5 in this chapter, with some updates, are in Sanchis et al. (2012).

## 4.1 Introduction

Any individual in modern societies –based on a consumption culture– faces a significant number of options when making any choice in the market. Examples can be easily found practically in every domain of daily life: a simple web search for a laptop produces hundreds of different product entries; shopping for a pair of jeans entails a deliberation among a surprisingly large number of different fits, waists, cuts, washes, zippers, etc; finding your favourite salad dressing seems a daunting task considering the huge variety offered in a regular supermarket. Being a consumer in the market seems quite costly in terms of choice.

As we already noted in previous chapters, psychological research has revealed that such an explosion of choice may affect consumer's welfare in a way that is contrary to a basic principle of market culture, namely "more choice implies more freedom which in turn implies more welfare". According to Schwartz (2000), this dogma is so deeply rooted in industrialized societies that in the end leads to a tyranny of choice paradoxically producing dissatisfaction rather than liberation. Schwartz suggested that beyond certain number of options more choice actually decreases satisfaction and termed this phenomenon the paradox of choice (Schwartz, 2005). He also claimed the existence of a paralysis effect, i.e. decision makers that have to face a very high number of options eventually do not solve the choice problem, choosing not to choose. This intriguing idea has been supported by some experiments or field studies, in particular by Iyengar and Lepper (2000). The fact that enlarging the choice set decreases the value of a welfare function apparently defies the logic of rational decision and entails interesting potential implications in theoretical microeconomics.

The cost of time has been implicitly considered as a contributing factor to the problems associated with choice overload. However, a formal model based on time use has not been proposed so far to address this issue in the literature. In chapter 3 we offer a model for this topic, which has been extensively discussed and analysed from a theoretical approach. This chapter is devoted to carry out numerical analysis of different cases derived from the model, in order to illustrate how the model behaves.

The numerical analysis is organized as follows. First, we describe some particular features used in the analysis. Second, we generate numerical values based on those features. Lastly, we show the results and discussions for all the cases considered, including a case study with actual data on prices in a specific problem of consumption. Altogether provides empirical evidence of the choice overload and the paradox of choice. The model can be numerically implemented for any specific choice problem provided that market data are available. Consumer search behaviour and product price structure are the basic inputs of the model. The main output of the numerical analysis is an optimal time distribution that provides the underlying structure of the rational solution of the choice problem. The analysis also yields as a by-product the number of product options –if any– that induce consumer's overload and discomfort. The ability of our analysis to explain both overload and choice distress is exhibited by considering both numerics from a theoretical uniform distribution for market prices and a naive case study for consumers planning a tour around Europe with actual data on prices. It is clear how to perform a similar analysis –*mutatis mutandis*– in any other choice problem with time use as the underlying resource.

## 4.2 The model

A consumer who faces any choice in the market implicitly decides about how to spend her total available mass of time ($T$) in three basic different uses of time: shopping time (employed in the search and decision about what to buy), working time, and free time (devoted to anything but shopping or working). The individual must fulfil the time constraint $T_b + T_c + T_l = T$, where $T_b$ is shopping time, $T_l$ is working time, and $T_c$ is free time.

The consumer typically finds a large number of market options for every product. Let us focus on her decision about acquiring a single product among the many versions of the product offered in the market. Let $N$ be the number of product options that she finds and inspects to make her buying decision. Her total expenditure is bounded from below by some quantity $G$ which is clearly a function of the number of options $N$, that is $G = G(N)$. The consumption problem is thus subject to the budget constraint defined by $G(N) \leq wT_l + V$, where $w$ is the wage rate per unit of working time ($T_l$), and $V$ is non-working income or savings. Since $G(N)$ represents the best deal for the searched product, it depends in a non-increasing fashion of the $N$ options of the product checked by the consumer.

Since the best price offer decreases as the number of seen options increases, there is incentive to look for more options and in turn to spend more time searching in the market. On the other hand, searching for more options entails a minimum shopping time which typically depends on the number of options. Let $\tau(N)$ denote the minimum shopping time that is necessary to find and evaluate $N$ versions of the product. The consumer problem must fulfil the time constraint $T_b \geq \tau(N)$. Notice that the shopping time floor defined by $\tau(N)$ may depend on the search efficiency of the consumer, and also on the organization of the market of the product. In general, it can be assumed that $\tau(N)$ is a non-decreasing function of $N$.

Under the standard assumption of rational behaviour, the consumer seeks to maximize her welfare, represented by $U(Z_1, Z_2, Z_3)$. Such utility function is defined over the wants which represent shopping satisfaction, personal satisfaction and job satisfaction, respectively described by $Z_1 = Z_1(T_b)$, $Z_2 = Z_2(T_c)$ and $Z_3 = Z_3(T_l)$. These wants are produced by the three uses of time, and for simplicity we rule out joint production.

Therefore, the consumer choice problem is modelled as the optimization problem described in (4.1), –rewritten below for convenience–, in which she determines the time distribution $(T_b, T_c, T_l)$ that maximizes her welfare function subject to time and budget constraints.

$$\begin{cases} \max\limits_{T_b,T_c,T_l} & U = U(Z_1(T_b), Z_2(T_c), Z_3(T_l)), \\ s.t. & G(N) \leq wT_l + V, \\ & T_b + T_c + T_l = T, \\ & T_b \geq \tau(N), \\ & T_c \geq 0, \\ & T_l \geq 0. \end{cases} \tag{4.1}$$

## 4.3 Data and methods

### 4.3.1 Constructing $G(N)$

In order to carry out the analysis, price estimates of the different product versions are needed to build the least-expenditure function $G(N)$ as a function of the number $N$ of alternatives in the market. The function $G(N)$ should be constructed specifically for each case study.

As a key theoretical case study, it can be assumed that prices are distributed uniformly on some interval $[\underline{p}, \overline{p}]$ and the set of the $N$ product versions that are explored is a simple random sample of prices. The function $G(N)$ is then naturally defined as

$$G(N) = E[\min(p_1, ..., p_N)], \tag{4.2}$$

where $E$ denotes the expectation and $Y = \min(p_1, ..., p_N)$ is the minimum of a simple random sample of i.i.d. prices $p_i \sim U([\underline{p}, \overline{p}]), i = 1, ..., N$.

This assumption puts together the price market structure of the searched product, which is assumed uniform here, and the consumer's behaviour, who faces a budget constraint where lower expenditure is given by the best expected price obtained from a random sample of the $N$ product versions considered.

Notice that any other underlying distribution may be considered for the prices of the product versions.

Besides being a natural option, the assumption of uniform distribution for prices has the advantage that the expression (4.2) can be expressed in a simple closed form in terms of $N$ and the support of the distribution. Indeed, we have

$$G(N) = \frac{\underline{p}N + \overline{p}}{N + 1}. \tag{4.3}$$

Notice that $G'(N) < 0$ for all $N$ and $G'(N) \to \underline{p}$ as $N \to \infty$. This implies that the expected lowest expenditure decreases as the size of the sample increases and, in the limit, the best expected price of a sample approaches the lowest possible price defined by the distribution.

Expression in (4.3) is obtained from the formula for the probability density function $h$ of the minimum in an i.i.d. sample of size $N$ for a random variable $X$, whose cumulative distribution function is $F(x)$ and probability density function $f(x)$. Specifically,

$$h(x) = N\,(1 - F(x))^{N-1}\,f(x). \tag{4.4}$$

This is obtained from the following simple calculation:

Let $H$ denote the cumulative distribution function of $Y_N = \min(X_1, ..., X_N)$, with $x_i$ i.i.d. and with support $[\underline{p}, \overline{p}]$. Let $y$ be arbitrary in $[\underline{p}, \overline{p}]$. Then:

$$\begin{gathered}
H(y) = P(\min(x_1, ..., x_N) \leq y) = 1 - P(\min(x_1, ..., x_N) > y) = \\
= 1 - P(x_1 > y, x_2 > y, ..., x_N > y) = \\
= 1 - P(x_1 > y) \cdot P(x_2 > y) \cdot ... \cdot P(x_N > y) = \\
= 1 - P(x_1)^N = 1 - (1 - F(y))^N.
\end{gathered}$$

Taking derivative with respect to $y$, we obtain formula (4.4) for the density $h$ of $Y$, as we wanted to show.

Now, let $Y_N = \min\{p_1, p_2, ..., p_N\}$ where $p_i$ is a random variable with cumulative distribution function $F(x)$ and probability density function $f(x)$.

Then for any underlying distribution of the random variables $p_i$ with support in $[\underline{p}, \overline{p}]$, it follows that integrating by parts we obtain:

$$\begin{aligned}
E[Y_N] &= \int_{\underline{p}}^{\overline{p}} xN\,(1 - F(x))^{N-1} f(x)dx \\
&= -\overline{p}\,(1 - F(\overline{p}))^N + \underline{p}\left(1 - F(\underline{p})\right)^N + \int_{\underline{p}}^{\overline{p}} (1 - F(x))^N\,dx \qquad (4.5) \\
&= \underline{p} + \int_{\underline{p}}^{\overline{p}} (1 - F(x))^N\,dx.
\end{aligned}$$

This formula gives the expectation of the minimum of an $N$-sample of a random variable with distribution $F$ supported in $[\underline{p}, \overline{p}]$ in terms of $\underline{p}, \overline{p}, F$ and $N$. Notice that in general we have $E[Y_N] \to \underline{p}$ as $N \to \infty$ and

$$\frac{\partial}{\partial N} E[Y_N] = \int_{\underline{p}}^{\overline{p}} (1 - F(x))^N \log\left(1 - F(x)\right) dx$$

from Leibniz's rule. It follows that $E[Y_N]$ is a decreasing function of $N$ that approaches the minimum left hand side of the interval supporting $F$. This holds for any underlying distribution $F$.

In the case that $p_i$ follows a uniform distribution $U([\underline{p},\overline{p}])$ on $[\underline{p},\overline{p}]$, we have that $f(x) = \dfrac{1}{\overline{p}-\underline{p}}, x \in [\underline{p},\overline{p}]$, $F(x) = \dfrac{x-\underline{p}}{\overline{p}-\underline{p}}, x \in [\underline{p},\overline{p}]$ and formula (4.5) gives

$$\begin{aligned}
G(N) = E[Y_N] \quad &= \underline{p} + \int_{\underline{p}}^{\overline{p}} \left(1 - \frac{x-\underline{p}}{\overline{p}-\underline{p}}\right)^N dx = \\
&= \underline{p} + \int_{\underline{p}}^{\overline{p}} \left(\frac{\overline{p}-x}{\overline{p}-\underline{p}}\right)^N dx = \\
&= \underline{p} + \left[\frac{\overline{p}-\underline{p}}{N+1}\left(\frac{\overline{p}-x}{\overline{p}-\underline{p}}\right)^{N+1}\right]_{\underline{p}}^{\overline{p}} \\
&= \underline{p} + \frac{\overline{p}-\underline{p}}{N+1} \\
&= \frac{\underline{p}N+\overline{p}}{N+1}.
\end{aligned}$$

This simple expression describes the lower bound of the budget constraint for the consumer's problem and consumer's behaviour when shopping in the market place among different options of a product whose price is uniformly distributed.

### 4.3.2 Constructing $\tau(N)$

The function $\tau(N)$ defining the shopping time floor (3.4) is obtained using textbook assumptions in economic theory. Strictly speaking, it is a cost function that maps the number of searches into its cost in terms of time. It is usually assumed in economics that the marginal cost of an additional search either is constant or increases with the number of searches. A general structure of $\tau(N)$ that allows for both assumptions is a first order stochastic difference equation:

$$\tau(N) = \tau(N-1) + \varepsilon(N) \quad N = 1, 2, ... \tag{4.6}$$

starting from some initial condition $\tau(0)$ and being $\{\varepsilon(N)\}_N$ some random sequence. Clearly, the marginal cost of the $N$-th search is precisely $\varepsilon(N)$, and thus the referred assumptions are easily translated into assumptions on the probability law for that random sequence. First, we claim that

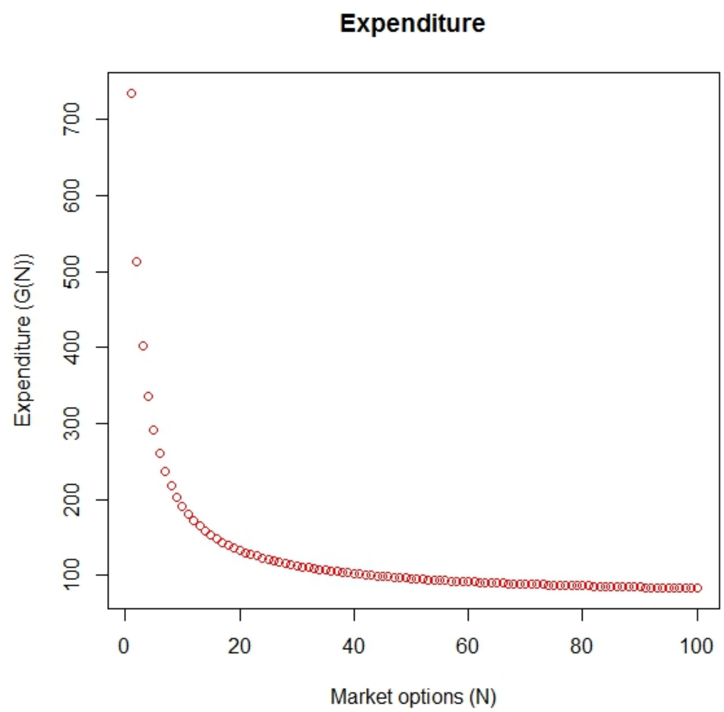


Figure 4.1: Expenditure $G(N)$ with respect to the number of market options $N$ with prices assumed to follow a uniform distribution on the interval $[70, 1400]$, generated by the equation 4.3.

a random nature for that marginal cost is fairly realistic. Second, it is also plausible that the marginal cost must be bounded from below by 0. Now, we model a constant marginal cost by taking for $\varepsilon(N)$ a probability distribution independent of $N$, whereas increasing marginal cost is modelled by considering a probability distribution whose expected value increases with $N$. We have used uniform distributions defined on some interval $[\underline{\varepsilon}, \overline{\varepsilon}]$ for the shock distribution. The initial value $\tau(0)$ is also uniformly distributed in some interval $[\underline{\tau}_0, \overline{\tau}_0]$ regardless of the marginal cost structure applied thereafter. Exploring alternative interval values as well as probability distributions produced no qualitative differences on the essential results, so we opted for the uniform distribution model for the sake of simplicity. In both cases, we generate a number of realizations of whole the sequence $\{\tau(N)\}_N$

and then we take the average sequence to be the search cost function of the problem. We have used 500 realizations for the final product.

### 4.3.3 Defining consumer preferences

We assume that the consumer's welfare is described in terms of time uses by a standard Cobb-Douglas function which is commonly used in economics (Mas-Colell et al., 1995; Varian, 1992), namely

$$U\left(Z_1, Z_2, Z_3\right) = a_1 \log\left(Z_1\right) + a_2 \log\left(Z_2\right) + a_3 \log\left(Z_3\right), \tag{4.7}$$

where $a_m$ are real parameters for $m = 1, 2, 3$, and of course $Z_1$ stands for shopping satisfaction, $Z_2$ is personal satisfaction, and $Z_3$ is job satisfaction.

Each of the wants or satisfactions $Z_m$ are obtained in two different ways, depending on whether we consider the analysis of main cases of the model or the case study with actual data on prices.

For the analysis of main cases, each $Z_m$ is also a standard Cobb-Douglas function with a constant term, in logarithmic form:

$$Z_1 = \log(1+T) + b_1 \log(1+T_b), \tag{4.8}$$

$$Z_2 = \log(1+T) + b_2 \log(1+T_c), \tag{4.9}$$

$$Z_3 = \log(1+T) + b_3 \log(1+T_l), \tag{4.10}$$

For the case study of travels around Europe we define each want as the corresponding time use variable:

$$Z_1 = T_b, \tag{4.11}$$

$$Z_2 = T_c, \tag{4.12}$$

$$Z_3 = T_l. \tag{4.13}$$

### 4.3.4 Optimization procedure

Once the functions $\{G(N)\}_N$ and $\{\tau(N)\}_N$ are computed, we continue with the optimization procedure. For a given set of parameter values (including $V$, $w$, $T$ and parameters $a_i$'s in the welfare function) and for each $N$ we solve the maximization problem described in the previous section using the well-known Broyden-Fletcher-Goldfarb-Shanno method, usually denoted in the literature as BFGS (Judd, 1998). Alternative methods show similar performance with regard to the optimal point at which convergence is achieved and computation time [2].

## 4.4 Main model cases

The cases under study in this section are those described theoretically in section 3.4 representing different consumer profiles, which we named ordinary consumers, workaholics, shopping lovers and unconstrained consumers. We will get through each case according to the model, data and methods introduced earlier.

In order to develop the analysis, we think of a simple example to illustrate the specific parameter values in the model. Let us consider a student who has to decide how to spend her time during the week ($T = 168$ hours). This student has a student loan from the past which generates a weekly payment of 125 EUR ($V = -125$ EUR)[3], and she also has to pay her consumption expenses. Those consumption expenses depend on the best deal she is able to find in the market, looking for the best price within the market options ($N$) for her consumption plan. Therefore, her expenditure in living expenses for the week is given by the function $G(N)$. This student must work in a student job at an hourly wage rate $w$ in order to cover her living expenses and debts, provided the accommodation in a student residence is already covered as a part of the student loan.

The distribution of her time generates certain satisfactions, which deter-

[2] The final version of the code was programmed in R, which allows for an easy integration of re-sampling methods ($G$), random number generation ($\tau$) and numerical optimization (BFGS is a built-in method). The final outcome was generated on an Intel Core i7, 2.80GHz PC, under Ubuntu 10.10. Both the code and the original data on prices are available from the author upon request.

[3] We consider here a consumer with debts, represented by the negative sign in savings, which of course do not depend on the market options scrutinized when shopping.

Table 4.1: Parametrical values for basic inputs in the model ($T$,$w$ and $V$), in the utility function ($U$) and in the expenditure function ($G(N)$)

| | Inputs | | | Utility | | | $G(N)$ | |
|---|---|---|---|---|---|---|---|---|
| Case | T | w | V | $a_1$ | $a_2$ | $a_3$ | $\underline{p}$ | $\overline{p}$ |
| Ordinary Consumer | 168 | 14 | -125 | 0.2 | 0.6 | 0.2 | 70 | 1400 |
| Workaholic | 168 | 14 | -125 | 0.2 | 0.4 | 0.4 | 70 | 1400 |
| Shopping Lover | 168 | 14 | -125 | 0.4 | 0.4 | 0.2 | 70 | 1400 |
| Unconstrained Consumer | 168 | 14 | -125 | 0.2 | 0.3 | 0.5 | 70 | 1400 |

mine her welfare, as defined in expressions (4.7), (4.8), (4.9) and (4.10). The choice problem is to maximize her welfare by deciding how to allocate her time during the week among working time, shopping time (to scrutinize the best deal in the market) and free time (to study, enjoy, rest and/or other things different from working and shopping). The optimal time allocation maximizes her welfare, given that this student must fulfil her budget constraint and the shopping time floor ($T_b \geq \tau(N)$).

Recall that market expenditure for living expenses ($G(N)$) for this student is assumed to be given by expression (4.3). The parameters $\underline{p}$ and $\overline{p}$ for $G(N)$ take the values presented in table 4.1, together with the parameter values for total week time in hours, wage rate per working hour and savings/debts ($T$, $w$ and $V$, respectively); parameter values $\underline{p}$ and $\overline{p}$ are thought as the cheapest deal in the market ($\underline{p}$ EUR) and the most expensive deal ($\overline{p}$ EUR) among all market options ($N$) for her consumption plan. Moreover, the expected shopping floor is defined by $\tau(N)$ according to (4.6). We expect as a fix time cost of shopping the average of $\underline{\tau_0}$ and $\overline{\tau_0}$ (e.g. going to the supermarket); similarly, the expected time to scrutinize each market option for her consumption plan is the average of $\underline{\varepsilon}$ and $\overline{\varepsilon}$; values for $\underline{\tau_0}$, $\overline{\tau_0}$, $\underline{\varepsilon}$ and $\overline{\varepsilon}$ are expressed in hours. All these parameter values are shown in table 4.2, along with the specific parameter values for the utility function which characterize each consumer profile.

Table 4.2: Parametrical values for the inputs in shopping satisfaction ($Z_1$), personal satisfaction ($Z_2$), job satisfaction ($Z_3$) and the shopping time floor ($\tau(N)$)

| | Satisfactions | | | Shopping time floor | | | |
|---|---|---|---|---|---|---|---|
| Case | $b_1$ | $b_2$ | $b_3$ | $\underline{\tau}_0$ | $\overline{\tau}_0$ | $\underline{\varepsilon}$ | $\overline{\varepsilon}$ |
| I (Ordinary Consumer) | -1 | 1 | -1 | 0 | 1 | 0 | 1 |
| II (Ordinary Consumer) | -1 | 1 | -1 | 0 | 1 | 0 | $e^{(0.10N)}$ |
| III (Ordinary Consumer) | -1 | 1 | -1 | 0 | 12 | 0 | 0 |
| IV (Workaholic) | -1 | 1 | 1 | 0 | 1 | 0 | $e^{(0.20N)}$ |
| V (Shopping Lover) | 1 | 1 | -1 | 0 | 1 | 0 | 0.1 |
| VI (Unconstrained Consumer) | 1 | 1 | 1 | 0 | 1 | 0 | 0 |

Table 4.3: Optimal solutions for shopping time ($T_b$), free time ($T_c$) and working time ($T_l$), values at the optimum for shopping satisfaction ($Z_1$), personal satisfaction ($Z_2$), job satisfaction ($Z_3$) and welfare ($W$), for all cases under analysis

| | Time allocations | | | Satisfactions and Welfare | | | |
|---|---|---|---|---|---|---|---|
| Case | $T_b^*$ | $T_c^*$ | $T_l^*$ | $Z_1^*$ | $Z_2^*$ | $Z_3^*$ | $W$ |
| I (Ordinary Consumer) | 3.99 | 139.53 | 24.48 | 3.52 | 10.07 | 1.89 | 1.76 |
| II (Ordinary Consumer) | 3.36 | 134.88 | 29.76 | 3.66 | 10.04 | 1.70 | 1.75 |
| III (Ordinary Consumer) | 5.97 | 147.91 | 14.12 | 3.19 | 10.13 | 2.41 | 1.80 |
| IV (Workaholic) | 0.48 | 83.76 | 83.76 | 4.74 | 9.57 | 9.57 | 2.12 |
| V (Shopping Lover) | 76.94 | 76.94 | 14.12 | 9.49 | 9.49 | 2.41 | 1.98 |
| VI (Unconstrained Consumer) | 35.20 | 51.12 | 81.67 | 8.72 | 9.08 | 9.54 | 2.22 |

### 4.4.1 Results and discussion

#### Description of cases

We analyse different cases both to illustrate the theoretical discussion in previous chapter and to demonstrate the robustness of our numerical findings. We find particularly interesting to consider different attitudes in regard to the search efficiency and to the sensitivity of welfare to time use. The following five cases illustrate all these points.

#### Case I: ordinary consumer with linear $\tau(N)$

We consider an individual who dislikes to increase working and shopping, according to the parameters of the three satisfactions in table 4.2, that is, we set $b_1 < 0, b_3 < 0$ and $b_2 > 0$. This case corresponds with that of a shopping time floor with constant marginal cost. Thus the individual spends a random time $\varepsilon(N)$ inspecting the $N$-th option which is independent of the number of options previously checked. We assumed that $\varepsilon(N)$ is uniformly distributed on the time interval $[0, 1]$, which amounts to spend half an hour on average exploring each considered consumption option in the market. The equation (4.6) previously described produces a function $\tau(N)$ with a linear shape, whose graph can be seen in figure 4.2.

#### Case II: ordinary consumer with convex $\tau(N)$

An individual who does not prefer to increase both working and shopping is also considered here. Preferences are as in case I above. In this case the shopping time floor has increasing marginal cost. To explore the $N$-th option, the consumer spends a random time $\varepsilon(N)$ whose expected value grows with the number of scrutinized options. This is modelled with an $\varepsilon(N)$ that follows a uniform distribution on an interval with increasing length with respect to $N$. In this exercise, we consider intervals whose length increases exponentially with $N$ (see table 4.2). Consequently, the expected time per option will also increase exponentially. The method in (4.6) produces here a convex and increasing shape for the $\tau(N)$ function. The resulting curve is displayed in figure 4.4. The convexity of the curve can be interpreted as a fatigue effect in the search activity.

Table 4.4: Market options thresholds and best deals ($G(N^*)$ and $G(\overline{N})$) in the market for all cases under analysis

| | Thresholds | | Best deals | |
|---|---|---|---|---|
| Case | $N^*$ | $\overline{N}$ | $G(N^*)$ | $G(\overline{N})$ |
| I (Ordinary Consumer) | 8 | 307 | 217.78 | 74.32 |
| II (Ordinary Consumer) | 5 | 34 | 291.67 | 108.00 |
| III (Ordinary Consumer) | 500 | - | 72.65 | - |
| IV (Workaholic) | 1 | 20 | 735.00 | 133.33 |
| V (Shopping Lover) | 500 | - | 72.65 | - |
| VI (Unconstrained Consumer) | 16 | - | 148.23 | - |

#### Case III: ordinary consumer with flat $\tau(N)$

Working time and shopping time are also distressing for this individual (see table 4.2). In this the case the shopping time floor has no marginal cost; the only cost is a fixed amount of time which is a random time $\tau_0$ uniformly distributed in $[0, 1]$, which amounts to half an hour (see table 4.2). Consequently, the expected time per option is constant. The method in (4.6) produces here flat shape for the $\tau(N)$ function. The resulting curve is displayed in figure 4.6. This constant and fixed time cost is interpreted as the time–independent from the market options– which this consumer can spend to scrutinize all options in the market.

#### Case IV: workaholic with convex $\tau(N)$

In this case we consider an individual who dislikes to increase shopping time, however, as a workaholic, prefers more working time. With respect to shopping time floor, this individual is as the one in the case II; therefore, this individual experiences certain fatigue after scrutinizing market options, as it can be observed in figure 4.8.

### Case V: shopping lover with linear $\tau(N)$

The case of shopping lovers refers to individuals who like more shopping time. In this particular case we consider that she does not enjoy working time. Shopping time floor is as the one in the case of ordinary consumer I, but with a lower marginal cost of time to search-and-decide, so this individual does not get tired after checking market options (see figure 4.10).

### Case VI: unconstrained consumer with flat $\tau(N)$

This is an individual who likes both working and shopping, besides free time. Shopping time floor is as in the case of ordinary consumers 3; therefore, this individual just invest a fixed amount of time shopping, which is independent from the market options. This generates the flat shape for $\tau(N)$ depicted in figure 4.12.

### Results

Figures 4.2 to 4.13 illustrate the findings for the cases I to VI, which we have discussed so far. The shopping time floors, defined by $\tau(N)$, are shown in red (triangles upwards) in figures 4.2, 4.4, 4.6, 4.8, 4.10, and 4.12 – notice that $\tau(N)$ does not change in the cases II, and IV on the one hand, and in the cases III and VI on the other hand. It follows from (3.5) that $T_l \geq \frac{G(N)-V}{w} > 0$ which imposes a lower bound for the working time, or a time ceiling for the sum $T_b + T_c$, that is $T_b + T_c \leq \gamma(N)$ for each $N$. The curves $\gamma(N) := T - \frac{G(N)-V}{w}$ are shown in green (squares) in figures 4.2, 4.4, 4.6, 4.8, 4.10, and 4.12, and coincide with the curve in blue (squares) representing $T - T_l^*$; this blue curve is not perceived in the pictures –except for the cases IV and VI– because it coincides with the green curve. Notice that the curve $\gamma(N)$ is the same in the cases II and V.

In figures 4.2, 4.4, 4.6, 4.8, 4.10, and 4.12, optimal time allocations versus the number of options are displayed as follows. For every number of options $N$, the black curve (triangles downwards) and the blue curve (squares) partition the total time $T$ (which is 168, the week time in hours) in the three time uses of the model: $T_b$ is given by the distance from the $x$-axis to the black curve, $T_c$ is given by the distance between the black curve and the blue curve, and $T_l$ is the distance between the blue curve and the line $T = 168$.

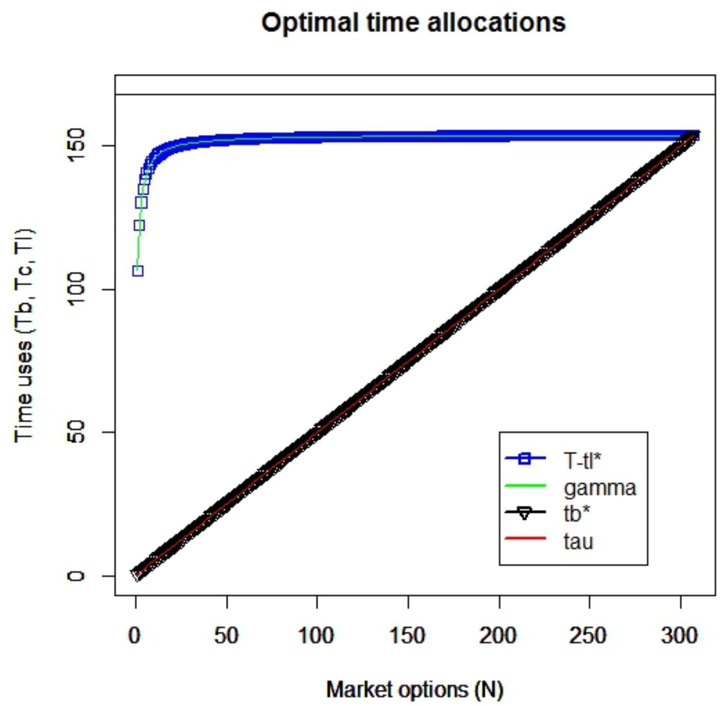


Figure 4.2: Optimal time allocation as a function of the number of options $N$ for an ordinary consumer whose expected search-and-checking time per option is constant (case I).

The corresponding curves representing the optimal values of welfare versus $N$ are displayed for all cases in figures 4.3, 4.5, 4.7, 4.9, 4.11, and 4.13. Two main outputs of the model analysis that are related with the choice overload problem can be seen in figures 4.2 to 4.13. First, notice that in cases I, II and IV there is some $\overline{N}$ such that $\tau(\overline{N}) = \gamma(\overline{N})$ –i.e. the point at which both curves intersect. It is clear from above that for $N > \overline{N}$ no feasible time distribution does exist. The values obtained for $\overline{N}$ in each case are shown in the first column of outputs in table 4.2. Secondly, in the cases I, II and IV it is apparent that the welfare function reaches its maximum at a certain value $N^*$, so that welfare increases while $N$ remains below $N^*$ but diminishes when the number of seen options is beyond $N^*$ (see figures 4.3, 4.5 and 4.9). The values obtained for this key output in each case are

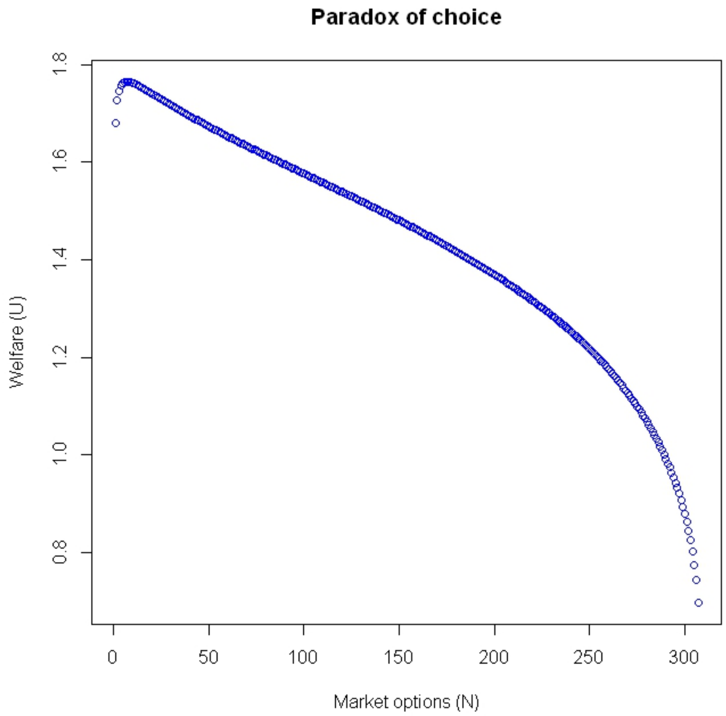


Figure 4.3: Welfare vs. $N$ for an ordinary consumer whose expected search-and-checking time per option is constant (case I).

displayed in the last column of table 4.2. For cases III, V and VI that is not the case: welfare never decreases with $N$ (see figures 4.7, 4.11 and 4.13). Notice that these could somehow be considered non-typical cases. What makes these consumers not suffering choice overload problem is some property that may not be expected to hold in general; namely a flat $\tau(N)$ (cases III and VI) and a strong preference for shopping (case V).

### Discussion

Our analysis clearly shows that a consumer may not check all the available options in the market in her search for a competitive deal. This is evidenced –except from the unconstrained consumers– in some typical cases

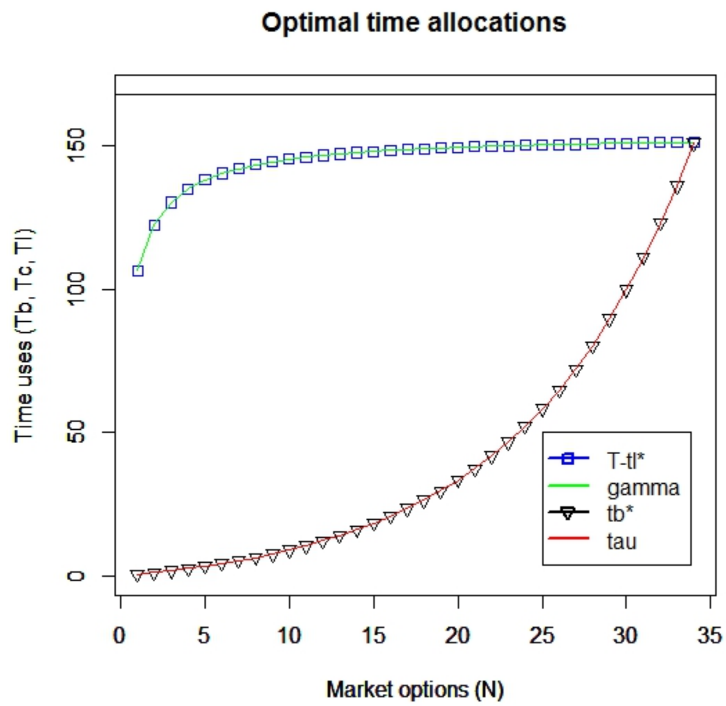


Figure 4.4: Optimal time allocation as a function of the number of options $N$ for an ordinary consumer who experiences fatigue when searching (case II).

under study, that is, with an increasing $\tau(N)$ and balanced time preferences among the uses of time. Pathological cases with flat $\tau(N)$ or strong preference for shopping may not display choice overload situations, namely paralysis effect or paradox of choice. The values of $\overline{N}$ for which feasible distributions no longer exists –cases I, II and IV– (see figures 4.2, 4.4 and 4.8) are well below the total number of available options in the market, which we set to 500. Although the expected price in the market would be reduced by searching beyond $\overline{N}$, the cost for the consumer in terms of time is overwhelming and she rejects looking for more consumption plans.

In concrete, given a welfare assessment, different profiles of shopping behaviour reduce feasibility from the total number of products (500) to

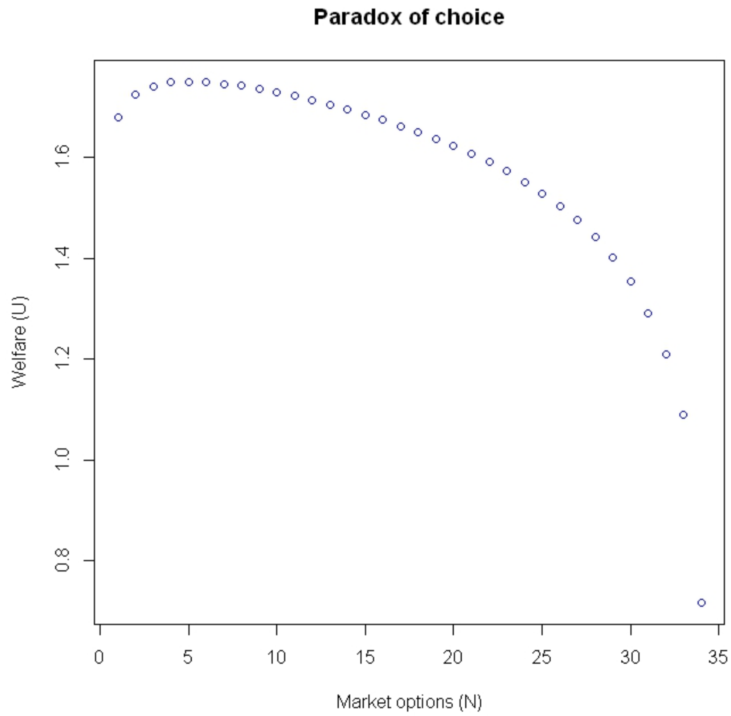


Figure 4.5: Welfare vs. $N$ for an ordinary consumer who experiences fatigue when searching (case II).

$\overline{N} = 307$ in the case I, who do not experience fatigue; when fatigue is considered in the search process, the choice overload is higher, reducing the number of options likely to be seen to $\overline{N} = 34$ (case II) and to $\overline{N} = 20$ (case IV). This is clearly linked to the choice overload problem. The number of options offered in the market typically surpasses the maximal number of options that a rational consumer is willing to explore, because she feels that the required time –both shopping and working– is just too demanding. The paralysis effect described by psychologists has its mathematical counterpart here in the emptiness of the feasible set of time distributions. Notice that this paralysis effect is a robust finding in the model analysis: its presence lies in the fact that a realistic $\tau(N)$ is increasing while $G(N)$ is typically decreasing.

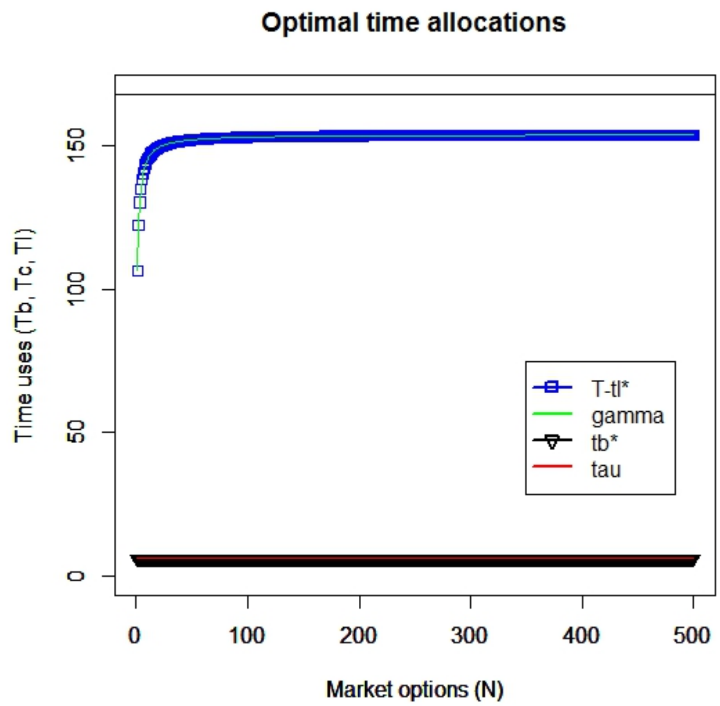


Figure 4.6: Optimal time allocation as a function of the number of options $N$ for an ordinary consumer whose expected search-and-checking time is constant and independent from the number of marker options (case III).

A typical consumer may abandon the choice problem with a number of options well below $\overline{N}$. This is due to another key pattern revealed by the analysis that occurs in the more typical cases (cases I, II and IV). It can be seen in figures 4.3 and 4.5 that welfare increases with the number of explored options, but only up to certain number $N^*$. Beyond $N^*$, welfare experienced by the consumer monotonically decreases (workaholics are the extreme case, since welfare decreases from the first option, as we observe in 4.9). In our numerical experiment, the number of options that triggers dissatisfaction drops from $N^* = 8$ in case I to $N^* = 5$ in case II (a 37.5% reduction), where the only difference is the fatigue when searching (in case IV we observe this threshold drops to $N^* = 1$, which is a dramatic fall). This finding certainly seems paradoxical, since the consumer is *doing better* as the number $N$ of

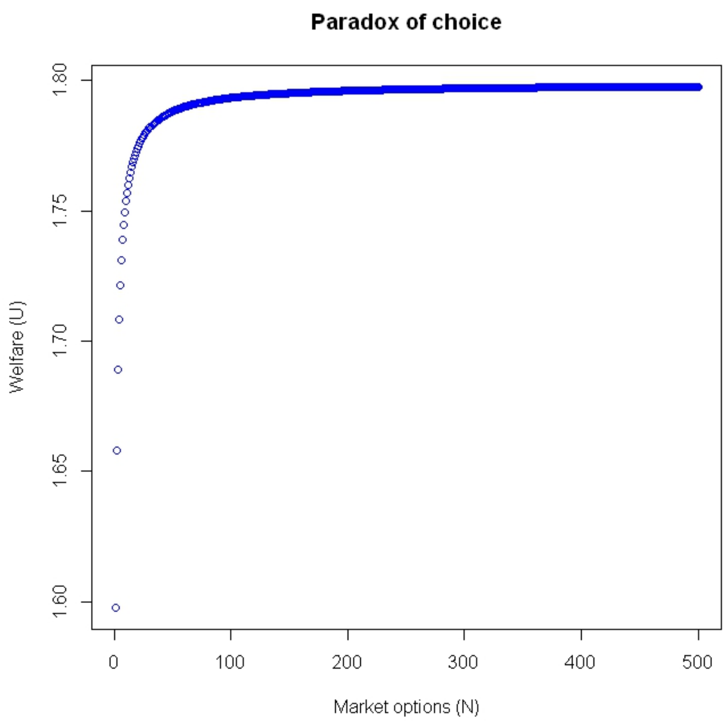


Figure 4.7: Welfare vs. $N$ for an individual whose expected search-and-checking time is constant and independent from the number of marker options (case III).

seen options increases –she is getting a better deal– *but* it turns out that she is *feeling worse*. This is the mathematical version of the so-called "paradox of choice" as formulated by social psychologist Schwartz (2005).

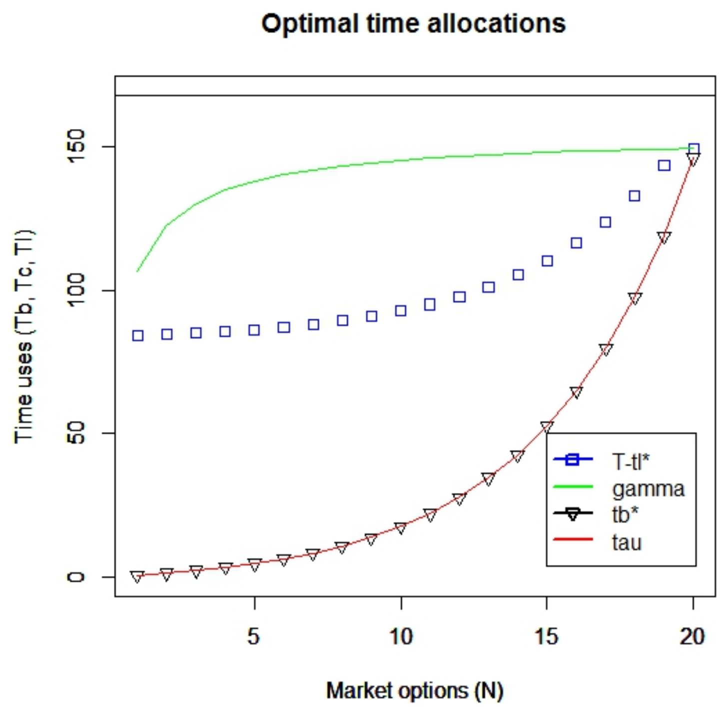


Figure 4.8: Optimal time allocation as a function of the number of options $N$ for workaholics whose expected search-and-checking time per option grows with $N$ (case IV).

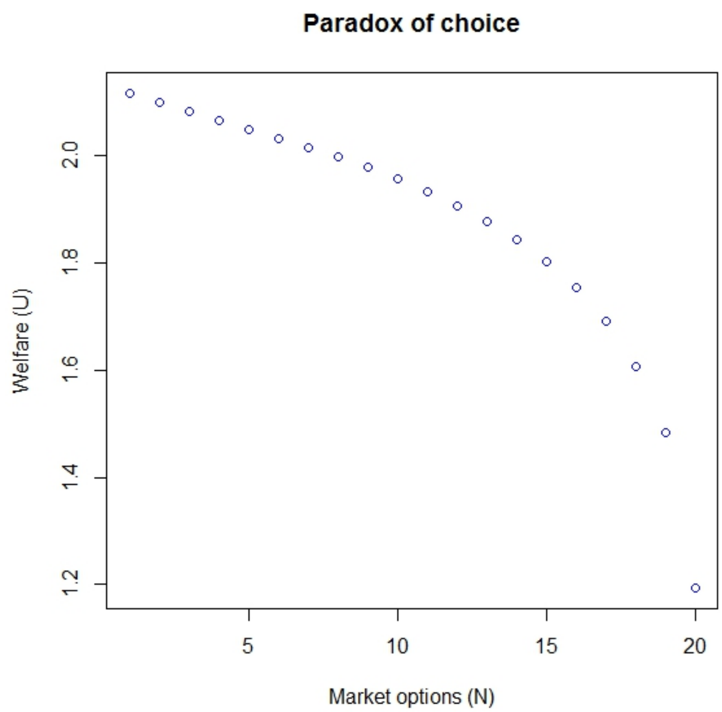


Figure 4.9: Welfare vs. $N$ for workaholics whose expected search-and-checking time per option grows with $N$ (case IV).

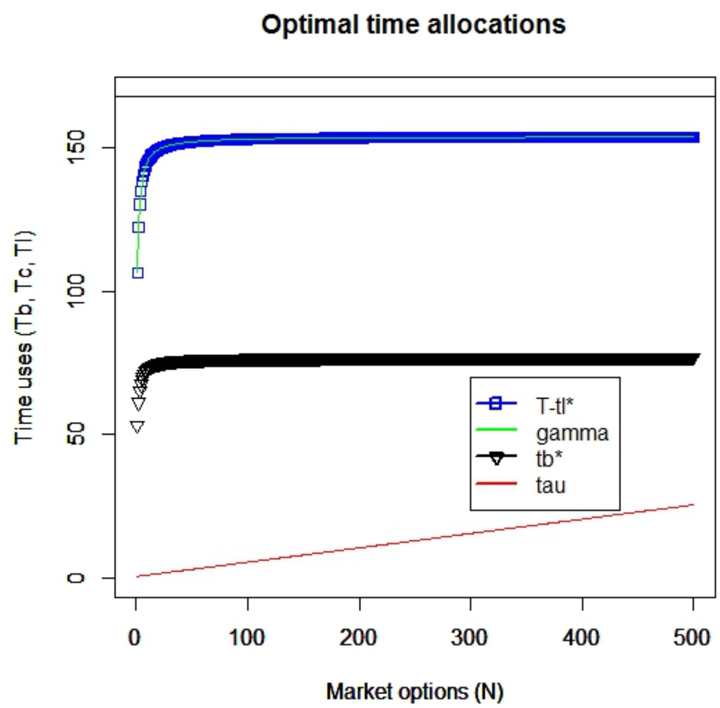


Figure 4.10: Optimal time allocation as a function of the number of options $N$ for a shopping lover (case V).

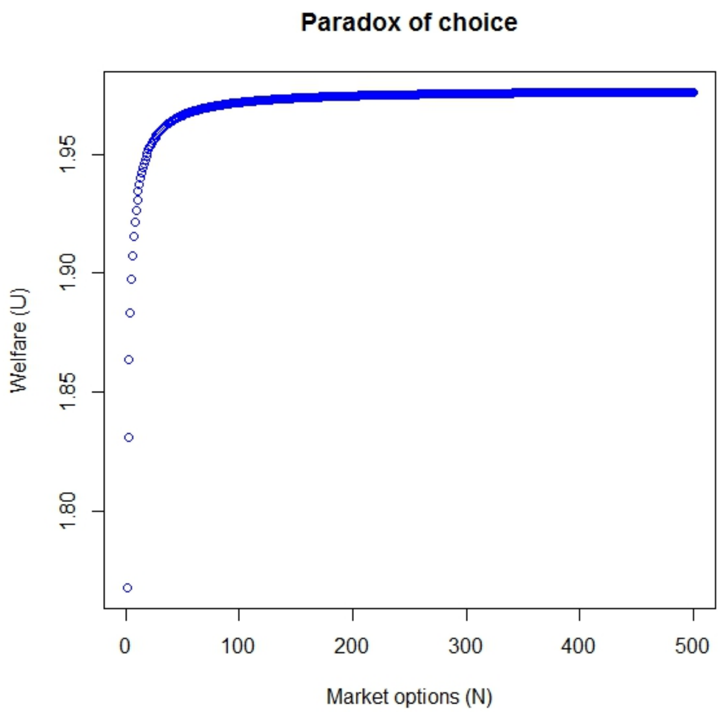


Figure 4.11: Welfare vs. $N$ for a shopping lover (case V).

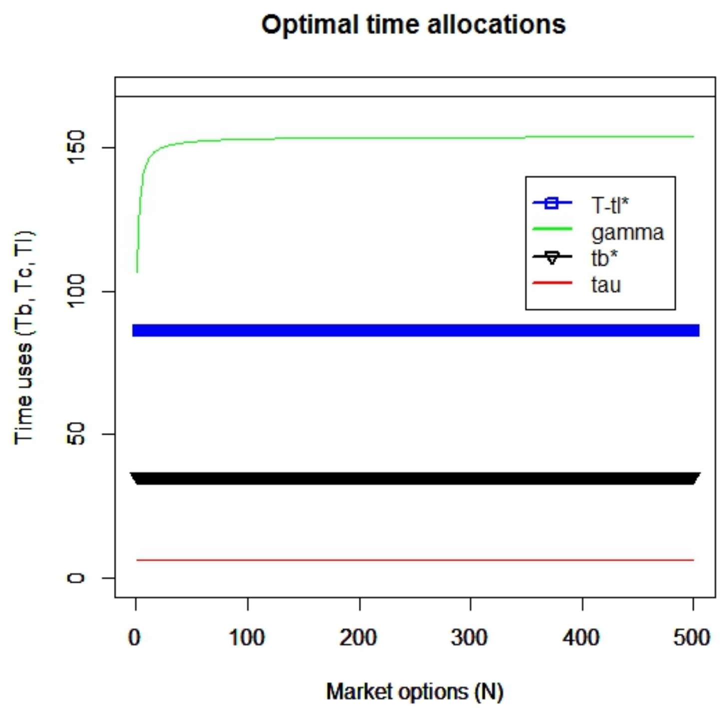


Figure 4.12: Optimal time allocation as a function of the number of options $N$ for an unconstrained consumer (case VI).

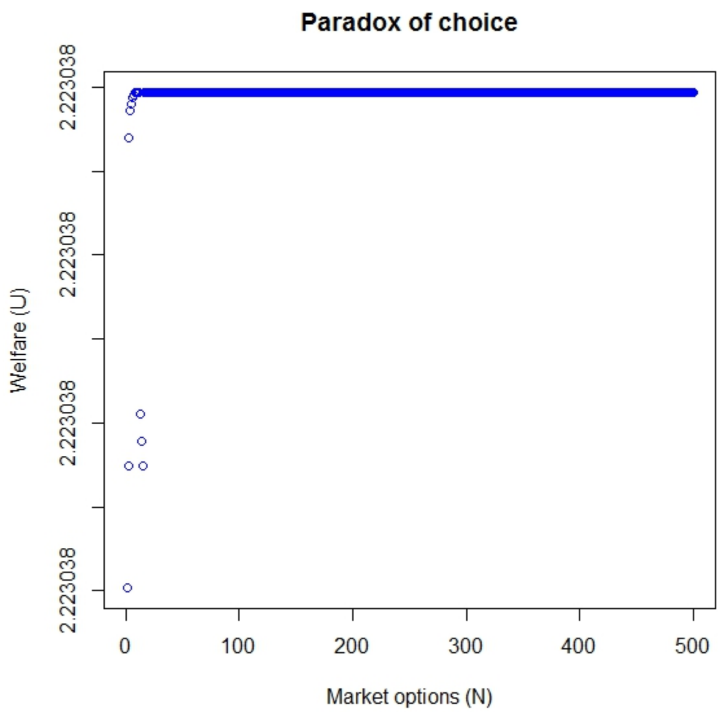


Figure 4.13: Welfare vs. $N$ for an unconstrained consumer (case VI).

## 4.5 A case study with data on prices

The framework described above is very general and it can be adapted to any search–and–buy situation in a multi–option market environment. The ability of the model to address the choice overload problem is illustrated with a simple case study, based on the situation described below.

A student is planning a vacation tour around Europe (at least two-weeks long) during June, in three months. She decides to look for an organized trip on the internet. A simple web search like "organized trips to Europe" produces thousands of pages with hundreds of entries each. So, even after filtering trip data in a suitable way, our student faces a choice problem with a huge number of versions for the product she is looking for. Furthermore, each travel that she finds feasible may require a significant amount of inspection time. Assume that she has got some small savings, say $V$, and that she can get extra money to buy the trip by working on weekends for a wage of $w$ per hour. All the money she earns is saved to pay for the trip. Since she is very busy during the week, her choice problem concerns weekend time only. She must decide how to allocate her total estimated weekend time $T$ during the spring term, given that she must work, search for a good deal on the internet, and enjoy herself the rest of the time.

For our particular simulation we use actual data on prices. A student is planning a vacation tour around Europe (at least two-weeks long) during June, in three months. She decides to look for an organized trip on the internet. A simple web search like "organized trips to Europe" produces thousands of pages with hundreds of entries each. So, even after filtering trip data in a suitable way, our student faces a choice problem with a huge number of versions for the product she is looking for. Furthermore, each travel that she finds feasible may require a significant amount of inspection time. Assume that she has got some small savings, say $V$, and that she can get extra money to buy the trip by working on weekends for a wage of $w$ per hour. All the money she earns is saved to pay for the trip. Since she is very busy during the week, her choice problem concerns weekend time only. She must decide how to allocate her total estimated weekend time $T$ during the spring term, given that she must work, search for a good deal on the internet, and enjoy herself the rest of the time. .

An enormous amount of data about tours around Europe can be obtained via an internet search. Our estimate of $G(N)$ is obtained by processing raw data just from a very popular US travel website, one that provides a wealth of information about prices of many tours around the world[4]. Filtering by "tours of two weeks length or more" we found 319 different products, and their corresponding prices are used to generate $G(N)$. We get our function $G(N)$ by using some re-sampling from the original data. The underlying idea is simple. Given a set of available prices, different consumers might perform searches in that set following different search rules. Even though all of them look for, say, the lowest price, they might look at prices in different order because of different search engines or search strategies. We try to capture the behaviour of an average consumer by re-sampling a number of times, so that each re-sample represents a consumer's path search, and then averaging across those re-samples.

Specifically, we proceed as follows. Let $\{p_1, ...., p_{\widehat{N}}\}$ be the set of real-life prices, where $\widehat{N}$ is arbitrary. We re-sample from that set by taking random permutations. Let us denote the $i$-th permutation by $\{p^i_{(1)}, ...., p^i_{(\widehat{N})}\}$. Thus, the $i$-th consumer perceives that the lowest price among the first $N$ options that she searched is $g_i(N) := \min\{p^i_{(1)}, ..., p^i_{(N)}\}$, for all $N \in \{1, ..., \widehat{N}\}$. We then take

$$G(N) := \frac{1}{I} \sum_i g_i(N)$$

where $I$ is the number of permutations. For the final version we have carried 500 permutations.

Notice that as $N$ increases, that is, when consumers have searched a large part of the dataset, all of them tend to agree on which is the lowest price of the sample. In the extreme case, $g_i(\widehat{N}) = \min\{p_1, ...., p_{\widehat{N}}\}$ regardless of the permutation index. However, at a time at which consumers have explored just a few prices, they might –and generally do– differ in their estimation on the lowest available price.

When processed as described above, our collected data on prices produces the graph of $G(N)$ that is represented in figure 4.14.

[4]Input data were obtained from the first entry listed by a popular internet search engine for the search term "organized trips to Europe", namely www.affordabletours.com. Travel data on this website were filtered by "tour destination to Europe", dates "june 2013", and length "14 days or more". A total number of 319 results were obtained (September 2012)

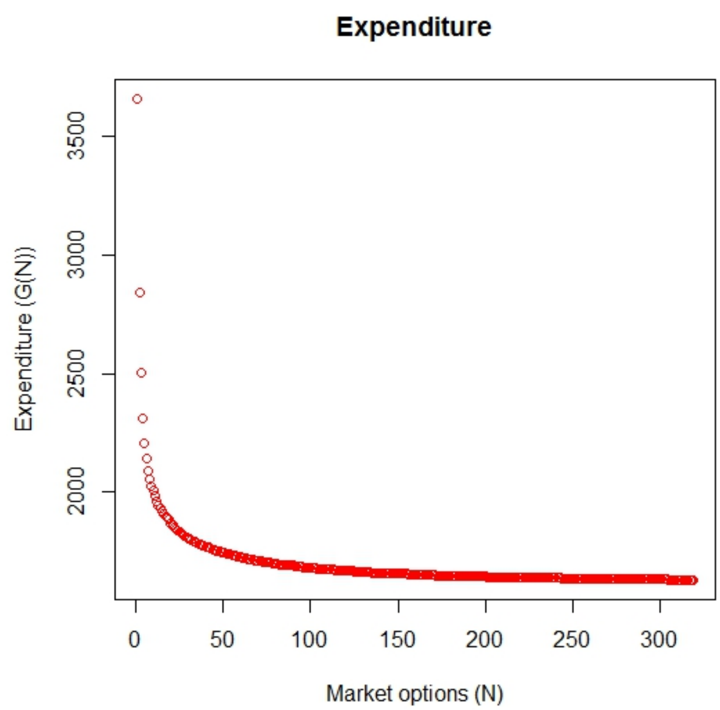


Figure 4.14: Expenditure $G(N)$ with respect to the number of market options $N$ for the case study with data on prices described in section 4.5.

For $\tau(N)$ we will keep to the method applied during this chapter, defined in (4.6).

### 4.5.1 Results and discussion

#### Description of cases

To demonstrate the robustness of our numerical findings, we analysed many different choice problems that are obtained by varying the model inputs. It is particularly interesting to consider different consumer attitudes, regarding either search efficiency or sensitivity of welfare to time use. Notice that satisfactions collapse to each time use, as we defined in (4.11), (4.12) and

(4.13); therefore, positive or negative preferences towards each use of time are represented by the parameters in the utility function. In particular, we consider the following standard Cobb-Douglas utility function in logarithmic form:

$$U(Z_1, Z_2, Z_3) = a_1 log(Z_1) + a_2 log(Z_2) + a_3 log(Z_3), \tag{4.14}$$

where $a_i, i = 1, 2, 3$ are the parameters defining the attitude towards different use of time.

The following three cases illustrate our study.

### Case #1: a consumer whose expected search-and-checking time per option is constant.

This case corresponds with a shopping time floor $\tau(N)$ with constant marginal cost. Thus the individual spends a random time $\varepsilon(N)$ inspecting the $N$-th option which is independent of the number of options previously checked. We assumed that $\varepsilon(N)$ is uniformly distributed on the time interval $[0, 2]$, which amounts to spend one hour on average exploring each considered trip option. The equation (4.6) produces a function $\tau(N)$ with a linear shape, whose graph can be seen in figure 4.15.

### Case #2: a consumer whose expected search-and-checking time per option grows with the number of seen options.

In this the case the shopping time floor has increasing marginal cost. To explore the $N$-th option, the individual spends a random time $\varepsilon(N)$ whose expected value grows with the number of seen options. As explained earlier in this chapter, this is modelled with an $\varepsilon(N)$ that follows a uniform distribution on an interval with increasing length with respect to $N$. In this exercise, we consider intervals whose length increases exponentially with $N$ (see table 4.5). As a consequence, the expected time per option will also increase exponentially. The method in (4.6) produces here a convex and increasing shape for the $\tau(N)$ function. The resulting curve is displayed in figure 4.17. The convexity of the curve is a consequence that the marginal time spent seeing each extra option increases with the number of seen options, which can be interpreted as a fatigue effect in the search activity.

**Case #3: a consumer who dislikes shopping.**

This case considers a significant variation in the features of the consumer of case #2, namely, her welfare is affected negatively with the shopping time. We thus assume that $a_1 < 0$ in this case (see table 4.5) so that her welfare in function (4.7) is a decreasing function of $T_b$. Also, wage rate is reduced by 40% with respect to cases #1 and #2. This will affect the individual's propensity to work and in turn her optimal time distribution compared to the individual in case #2.

The values for the parameters in each case are summarized in the table 4.5 below.

Table 4.5: Parametrical values for the inputs of the model analysis in the case study with actual price data

| | Inputs | | | Utility function and shopping floor | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Case | T | w | V | $a_1$ | $a_2$ | $a_3$ | $\underline{\tau}_0$ | $\overline{\tau}_0$ | $\underline{\varepsilon}$ | $\overline{\varepsilon}$ |
| 1 | 300 | 20 | 300 | 0.25 | 0.50 | 0.25 | 0 | 20 | 0 | 2 |
| 2 | 300 | 20 | 300 | 0.25 | 0.50 | 0.25 | 0 | 20 | 0 | $2e^{(0.02N)}$ |
| 3 | 300 | 12 | 300 | -0.05 | 0.50 | 0.25 | 0 | 20 | 0 | $2e^{(0.02N)}$ |

Table 4.6: Main outputs of the model analysis in the case study with actual price data. Optimal solutions for time allocations and welfare, and key threshold values for choice overload

| | Time allocations and Welfare | | | | Thresholds | |
|---|---|---|---|---|---|---|
| Case | $T_b^*$ | $T_c^*$ | $T_l^*$ | $W$ | $\overline{N}$ | $N^*$ |
| 1 | 65.31 | 159.69 | 75 | 4.664062 | 223 | 56 |
| 2 | 66.96 | 158.04 | 75 | 4.664062 | 84 | 38 |
| 3 | 16.87 | 133.98 | 149.15 | 3.558774 | 74 | 7 |

The primary output of the model is the optimal time allocation for each number of options $N$. Table 4.6 also shows two key outputs of the model ($\overline{N}$ and $N^*$) related with the choice overload problem that will be discussed below.

**Results**

Figures 4.15, 4.16, 4.17, 4.18, 4.19 and 4.20 summarize the model findings for case #1, case #2 and case #3 described in this section. The shopping time floors, defined by $\tau(N)$, are shown in red (triangles upwards) in figures 4.15, 4.17, and 4.19 –notice that $\tau(N)$ does not change in cases #2 and #3. It follows from (3.5) that $T_l \geq \max\{0, \frac{G(N)-V}{w}\}$ which imposes a lower bound for the working time, or a time ceiling for the sum $T_b + T_c$, that is $T_b + T_c \leq \gamma(N)$ for each $N$. The curves $\gamma(N) := \max\{0, T - \frac{G(N)-V}{w}\}$ are shown in green (squares) in figures 4.15, 4.17, and 4.19. Notice that the curve $\gamma(N)$ is the same in cases #1 and #2.

In figures 4.15, 4.17 and 4.19, optimal time allocations versus the number of options are displayed as follows. For every number of options $N$, the black curve (triangles downwards) and the blue curve (squares) partition the total time $T$ (which is 300 in our study) in the three time uses of the model: $T_b$ is given by the distance from the $x$-axis to the black curve, $T_c$ is given by the distance between the black curve and the blue curve, and $T_l$ is the distance between the blue curve and the line $T = 300$.

The corresponding curves representing the optimal values of welfare versus $N$ are displayed for the three cases in figures 4.16, 4.18, and 4.20. Two main outputs of the model analysis that are related with the choice overload problem can be seen in figures 4.15, 4.17, and 4.19. First, notice that in all cases there is some $\overline{N}$ such that $\tau(\overline{N}) = \gamma(\overline{N})$ –i.e. the point at which both curves intersect. It is clear from above that for $N > \overline{N}$ no feasible time distribution does exist. The values obtained for $\overline{N}$ in each case are shown in table 4.6. Secondly, in all cases it is apparent that the welfare function reaches its maximum at a certain value $N^*$, so that welfare increases while $N$ remains below $N^*$ but diminishes when the number of seen options is beyond $N^*$. The values obtained for this key output in each case are displayed in the last column of table 1.

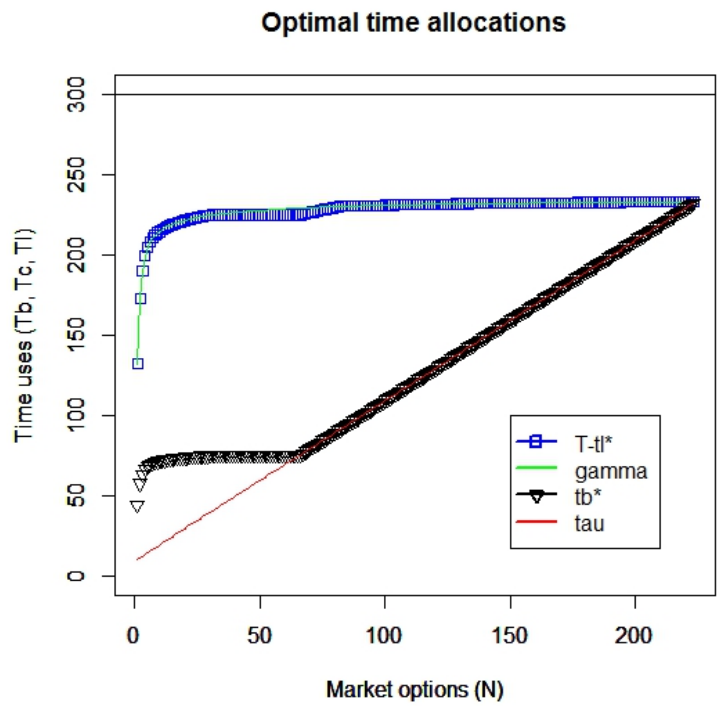


Figure 4.15: Optimal time allocation as a function of the number of options $N$ for an individual whose expected search-and-checking time per option is constant (case #1 in section 4.5.1).

### Discussion

Our analysis clearly shows that a consumer may not check all the available options in the market in her search for a competitive deal. This is evidenced in the three cases under study: the values of $\overline{N}$ for which feasible distributions no longer exists are well below the total number of available options in the market, that is 319. Although the expected price in the market would be reduced by searching beyond $\overline{N}$, the cost for the consumer in terms of time is overwhelming and she discards looking for more trips.

In particular, given a welfare valuation, different profiles of shopping behaviour reduce feasibility by a 44% (from $\overline{N} = 223$ to $\overline{N} = 84$ over 319), due to the effect of fatigue in searching. This fact is illustrated by cases #1 and

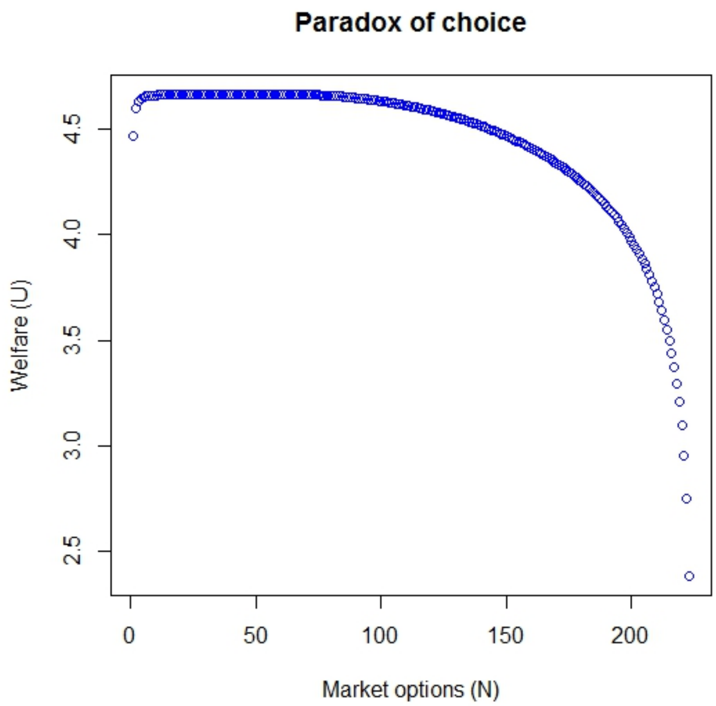


Figure 4.16: Welfare vs. $N$ for an individual whose expected search-and-checking time per option is constant (case #1 in section 4.5.1).

#2 above. Also, a shopping behaviour subjected to fatigue may reduce the number of feasible time distributions in the case that time spent shopping affects welfare negatively, what can be seen by comparing cases #2 and #3. In case #3, the maximal number of feasible distributions is lowered further to $\overline{N} = 74$. This is clearly linked to the paralysis in the phenomenon of choice overload problem. The number of options offered in the market typically surpasses the maximal number of options that a rational consumer is willing to explore, because she feels that the required time –both shopping and working– is just too demanding. The extreme paralysis described by psychologists has its mathematical counterpart here in the emptiness of the feasible set of time distributions. Notice that this paralysis effect is a robust finding in the model analysis: its occurrence lies in the fact that a realistic $\tau(N)$ is increasing while $G(N)$ is typically decreasing.

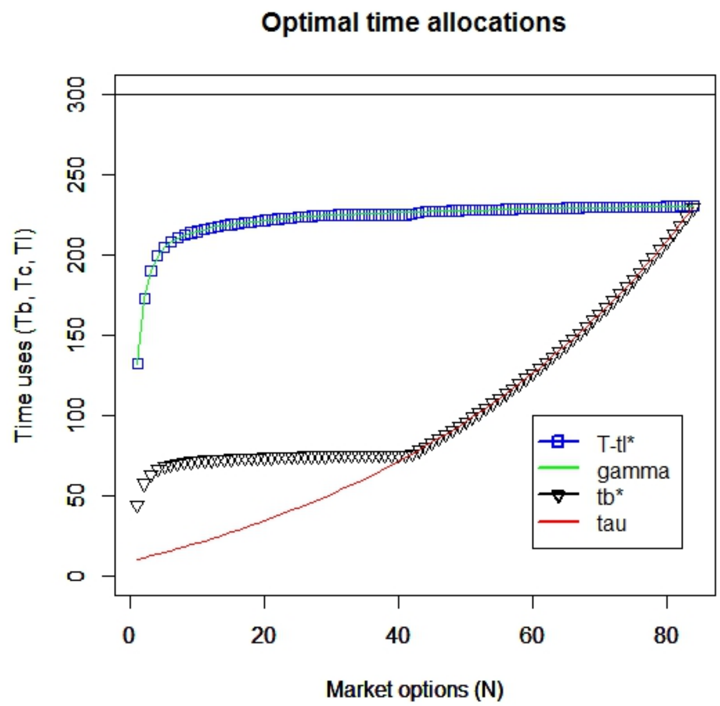


Figure 4.17: Optimal time allocation as a function of the number of options $N$ for an individual whose expected search-and-checking time per option grows with $N$ (case #2 in section 4.5.1).

It is expected that a typical consumer actually will decide to abandon the choice problem with a number of options well below $\overline{N}$. This is due to another key pattern revealed by the analysis that is repeated in every case. It can be seen in figures 4.16, 4.18, and 4.20 that welfare increases with the number of explored options, but only up to certain number $N^*$. Beyond $N^*$, welfare experienced by the consumer monotonically decreases. In our numerical experiment, the number of options that triggers dissatisfaction drops from $N^* = 56$ in case #1 to $N^* = 38$ in case #2. In case #3, this threshold drops to $N^* = 7$, which is a dramatic fall. This finding certainly seems paradoxical, since the consumer is doing better as the number $N$ of seen options increases –she is getting a better deal– but it turns out that she

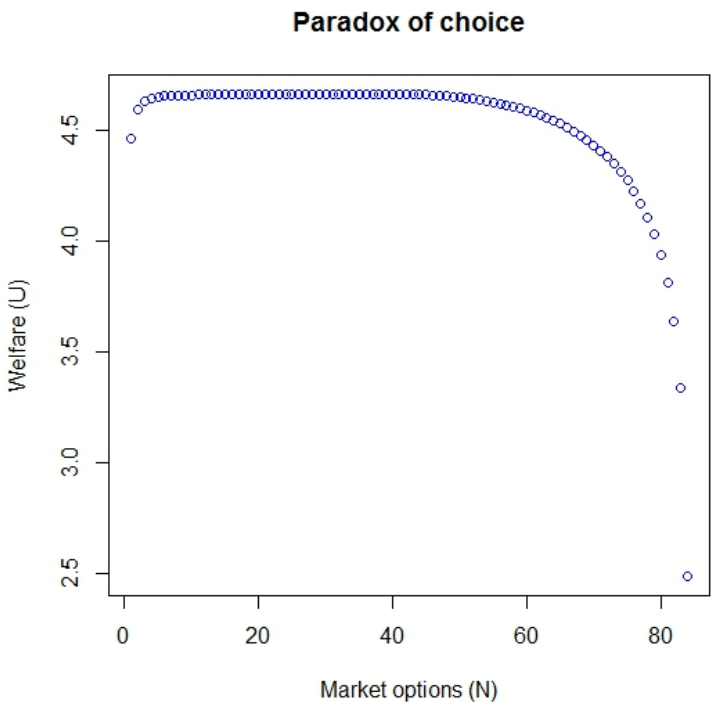


Figure 4.18: Welfare vs. $N$ for an individual whose expected search-and-checking time per option grows with $N$ (case #2 in section 4.5.1).

is feeling worse[5]. This is the mathematical version of the so-called "paradox of choice" as formulated by social psychologist Schwartz (2005).

## 4.6 General conclusions

We provide a flexible model of search-and-buy behaviour based on a rational allocation of time. Three different uses of time are considered –shopping time, leisure time and working time– to address the pervasive problem of a

[5] Best deals in the three cases are as follows. Case #1: $N^* = 56$ and $G(56) = 1730.99\$$ after investing $T_b = 65.31$h searching, spending $T_l = 75$h working and enjoying herself the remaining time. Case #2: $N^* = 38$ and $G(38) = 1774.12\$$ for $T_b = 66.96$h and $T_l = 75$h. Case #3: $N^* = 7$ and $G(7) = 2089.79\$$ for $T_b = 16.87$h and $T_l = 149.15$h.

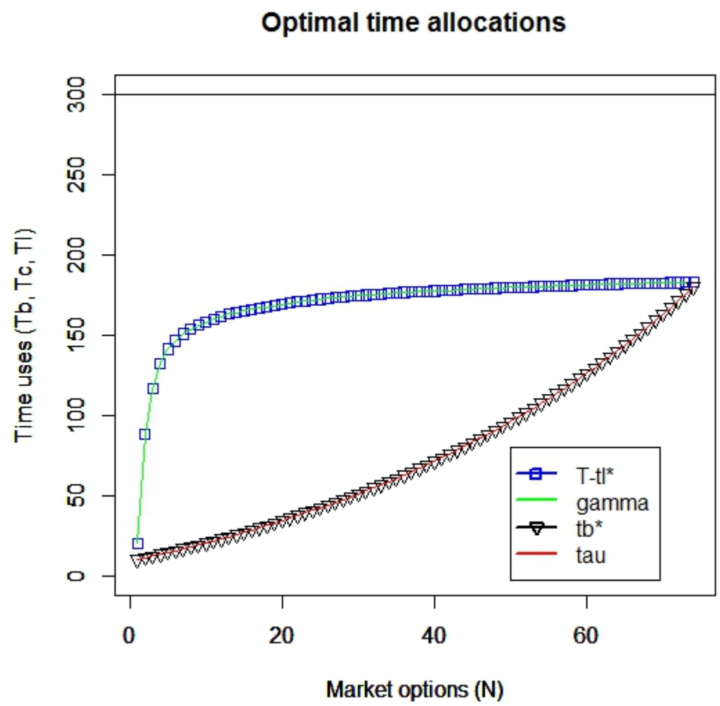


Figure 4.19: Optimal time allocation as a function of the number of options $N$ for a consumer who dislikes shopping (case #3 in section 4.5.1).

consumer that faces a huge number of market options and can get better deals by investing more time searching. The analysis produces an optimal time allocation in terms of the number of the considered options.

The numerical analysis of both the model cases for a week in the life of certain student and the model in a simple case study based on market prices of organized trips around Europe reproduces key features of consumers psychology when facing a shopping decision in a market with a vast number of weekly consumption plans. The analysis supplies a mathematical formalism for two specific issues raised by psychological research on choice overload, namely the paradox of choice –"more options imply less satisfaction"– and the paralysis effect –"the consumer decides not to decide". The model analysis also provides estimates for the number of options that trigger both issues.

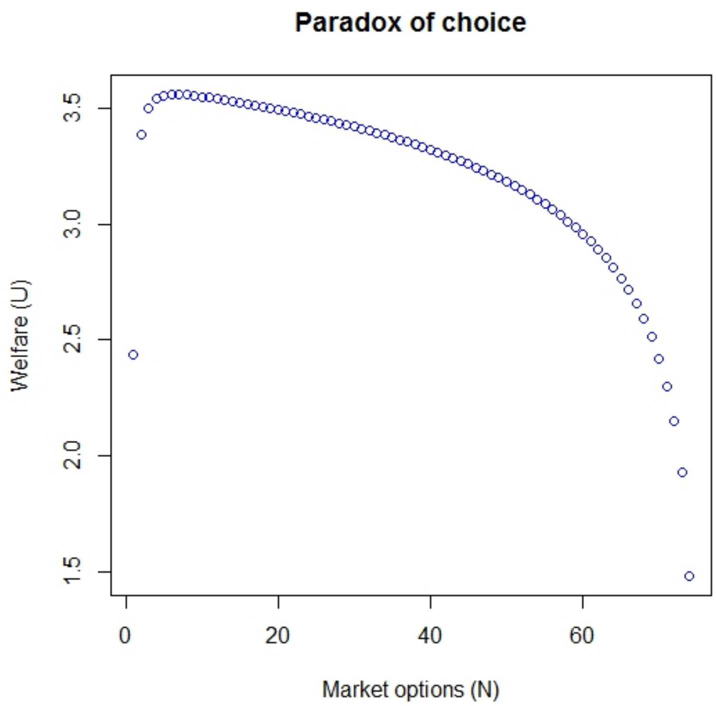


Figure 4.20: Welfare vs. $N$ for a consumer who dislikes shopping (case #3 in section 4.5.1).

# Chapter 5

# Summary and conclusions

> *It is not that we have so little time but that we lose so much. [... The time] we receive is not short but we make it so; we are not ill provided but use what we have wastefully.*
>
> (Lucius Annaeus Seneca)

In this dissertation we have introduced several improvements in theoretical and applied time microeconomics, that aim at an economic analysis including the use of time as a key variable. Below we offer a summary of our findings, along with the main conclusions and future research that can be derived from our research.

Microeconomic theory has paid little attention to models which include time use as a key explanatory variable. A basic example is the textbook leisure model in microeconomics. This simple setting provided valuable insights, for example, in labour economics. Time allocation models started by Gary S. Becker attracted the attention to this topic. We show in chapter 2 how time allocation models are a significant improvement and generalization of leisure models. However, these models entail some new problems, namely the absence of joint production and the need of constant returns to scale in the household production technology, i.e. in the production of wants. Later on in chapter 2, we provide an extended model about the allocation of time which allows for joint production both in time and goods. Such a big structure is illustrated in this chapter with one example. We reduce the extended model to analyse the effect of the extension in the retirement age in a lifetime context; this simple illustration yields some paradoxical effects policy makers should consider when evaluating any change concerning retirement age.

In chapter 3 we reduce the extended model to a simple case in which only time use variables are regarded in order to analyse possible choice overload situations. We initially refer to a choice problem concerning just one product with different versions or options in the market (different brands, etc). An individual must decide how to allocate her total time into three uses of time: shopping time, free time and working time. More shopping time allows the individual to find better deals in the market. Given the budget constraint and the time constraints, this individual must decide the time distribution which maximizes her welfare. A simple model like this generates different solutions, which we analyse in four cases: ordinary consumers, workaholics, shopping lovers and unconstrained consumers. From the discussion of these main cases we provide formal explanation to some choice overload phenomena described by social psychologists namely the paradox of choice and the paralysis effect. Mathematical conditions which guarantee these are provided.

Empirical findings for the theoretical model are illustrated by means of numerical analysis in chapter 4. Considering the model in chapter 3, we define the specific setting in order to develop our numerical analysis. We provide a method in order to generate both the price structure in the market and the shopping behaviour, and we apply a common optimization algorithm to calculate optimal solutions for several cases. The analysis is dual: on the one hand we consider a theoretical statistical distribution for prices in the market, and on the other hand we use actual data on prices. For these two situations, we first apply the model analysis to the main cases discussed in chapter 3, named as ordinary consumers, workaholics, shopping lovers and unconstrained consumers; secondly, we use the model to analyse realistic consumer profiles with actual data on prices, which is in accord with the theoretical cases. In all cases, we observe how the results in our numerical analysis are in line with the theoretical predictions in chapter 3.

General conclusions from the research in this dissertation are as follows. An extended model about the allocation of time which takes into account joint production is provided and its use is illustrated with an application concerning the retirement age. By applying our time microeconomic model to the study of choice overload situations, we learn the conditions under which several phenomena arise. Particularly, the typical price structure of a market product and our standard shopping behaviour could be sufficient to contradict a deeply rooted paradigm in our modern economies and societies: the more options to choose in the market, the better. We show how welfare will eventually decrease beyond some threshold level of choice in market options, which is known as the paradox of choice. Even though this paradox may not occur, we demonstrate that most likely we as consumers experience choice overload, i.e. we do not see all options in the market. It is also showed why sometimes consumers choose not to choose, the so called paralysis effect. The paradox of choice and the paralysis effect are successfully tested by our findings in the numerical analysis.

## Future research

Chapter 2 suggested the possibility of studying an extended version of the time use model that have been used to illustrate the effects of policies in favour of the increase in retirement age; the idea was to consider more individuals (the society) and to introduce fiscal aspects in order to study the different redistributions and effects which different pension systems generate. Moreover, to provide a dynamic version of the extended theory is another interesting and challenging research area for the future. Last, from this chapter, would be the challenge of generating a general equilibrium theory about the allocation of time; this would help to analyse more rigorously the efficiency in the use of time within a given economy.

Chapter 3 leaves several extensions of the model as future research. We highlight here the priority which we will give to the extension of the model that allows the consumer to choose endogenously the number of options out of all the given market options. This model would eventually contribute to the research in line with Iyengar and Kamenica (2010), Kamenica et al. (2011) and Caplin et al. (2012); this research suggests that increasing the number of options, not only reduces the participation in the market and reduces the welfare of consumers, but also affects to the type of options that are chosen. Another interesting effect to be studied in more detail is the categorization effect; future research in line with finding formal conditions for our model which explains the categorization effect would be a valuable finding. Considering a setting with more products, each of them with different market versions as options, would be interesting in order to study the presence or not of cross choice overload effects —or in contrast, synergies– when choosing among different products; this last task may relate quite much with the categorization effect at some stages. Lastly, to provide an alternative measurement of welfare which is based in time distributions or allocations is a challenging task we have already started, but due to its complexity and to be at a very preliminary stage it has not been included in this dissertation.

Chapter 4 and the numerical analysis we carried out has suggested us the possibility of doing something similar whenever we have finished the extensions of the model in chapter 3, as well as considering other underlying distributions instead of the uniform for the price structure. Moreover, by doing this numerical analysis we got the idea, mentioned above, about generating an alternative measurement of welfare which is based in time distributions.

# Chapter 6

# Introducción, resumen y conclusiones

*No es que tengamos poco tiempo, sino que perdemos mucho*

(Lucius Annaeus Seneca)

## Motivaciones y objetivos

*La economía es la ciencia que estudia el comportamiento humano como una relación entre los fines y los medios escasos que presentan diferentes usos alternativos* (Robbins, 1932). Todos podríamos estar de acuerdo en que nuestro principal recurso, en el sentido más estricto de la aplicación de la definición de economía, es el tiempo del que disponemos. El (escaso) tiempo que se nos otorga es probablemente el recurso con más usos alternativos para perseguir nuestros propósitos y fines.

Es destacable cómo la importancia que tiene el tiempo y su uso ha sido subestimada en el campo de la teoría microeconómica. Por ello, esta tesis responde a dicha percepción, y pretende reforzar esta importancia. Intentamos hacerlo resolviendo algunos problemas presentes en la literatura sobre el uso del tiempo y su asignación; asimismo, utilizamos el tiempo como un medio para explicar otros fenómenos de interés introducidos por hallazgos de investigación realizados en otras disciplinas, como es el caso de las situaciones de sobrecarga en la elección que experimentan los individuos que se enfrentan a un problema de elección.

En concreto, el objetivo principal de esta tesis doctoral es doble: primero, formular un modelo ampliado sobre el uso del tiempo que resuelva el problema de la producción conjunta presente en la literatura económica; segundo, aplicar este modelo para explicar formalmente las situaciones de sobrecarga en la elección experimentada por los consumidores. Como consecuencia de este doble objetivo, un nuevo objetivo que se deriva es el siguiente: tercero, mostrar validación empírica que contraste los resultados teóricos obtenidos que explican la sobrecarga en la elección. Estos son los tres objetivos de esta tesis doctoral.

## Revisión de la literatura

### Sobre el uso del tiempo y la teoría microeconómica

La teoría económica no había prácticamente considerado el tema del uso del tiempo antes de la década de 1960. Las primeras menciones a un argumento basado en el uso del tiempo las encontramos en Reid (1934) y en Mincer (1962). Sin embargo, ningún modelo era capaz de incluir las ideas que giraban en torno al uso del tiempo y la economía hasta dicha década.

Paralelamente, algunos otros economistas habían estado pensando en un nuevo modelo de enfocar la conducta microeconómica, en contraposición al modelo económico standard[1]. Por consiguiente, encontramos una modelización teórica en la tesis doctoral de Duncan Ironmonger, defendida en la Universidad de Cambridge en 1962, aunque publicada una década después, en 1972. Una modelización similar la encontramos en Lancaster (1966). No obstante, ni Ironmonger (1972) ni Lancaster (1966) se refieren explícitamente a los insumos de tiempo como parte central de sus modelos[2]; dichos modelos consideran un problema de maximización de utilidad, la cual está definida sobre lo que ellos llaman *commodities* o *wants*, que se obtienen usando diferentes insumos o a través de ciertas características. Fundamentalmente, Ironmonger (1972) argumenta –*grosso modo*– que para producir

[1]Basado en la maximización de la utilidad definida sobre los bienes de mercado, supuesto que el consumidor tiene recursos limitados y los bienes de mercado tienen un precio bien definido en un mercado competitivo.

[2]Reid (1934) es considerada como la génesis de dicha línea de pensamiento encaminada a un nuevo modelo de enfocar la conducta microeconómica y también es considerada como un antecedente directo de Becker (1965). Sin embargo, el trabajo de Margaret Reid no ha sido suficientemente considerado por buena parte de la literatura económica.

cada *want* se necesitan algunos *inputs* o insumos, mientras que Lancaster (1966) considera que cada bien o *commodity* es diferente si las características que presenta cada bien difieren.

La particularización del modelo en Ironmonger (1972) que considera variables de uso del tiempo llega con Becker (1965) y su *teoría de la asignación del tiempo*. Becker (1965) arguye como veremos posteriormente que los consumidores maximizan una utilidad definida sobre lo que Becker (1965) denomina *commodities*; tales *commodities* se producen a través del uso de bienes de mercado y tiempo empleado para su producción, y en consecuencia el consumidor se enfrenta a restricciones tanto presupuestarias como de uso del tiempo.

Becker (1965) ha generado un campo de investigación muy extenso, particularmente en economía y sociología. Sin embargo, las contribuciones teóricas presentes en la teoría económica han sido escasas desde este artículo. Entre la literatura que intenta contribuir a la mejora de la teoría económica hasta la fecha en este campo, encontramos algunos modelos destacables como DeSerpa (1971) o Evans (1972), aunque estos eran esencialmente casos particulares deBecker (1965). Más interesantes fueron las duras críticas a Becker (1965) que se encuentran en Pollak and Wachter (1975). Tales críticas, como veremos en el capítulo 2, muestran los principales problemas de los modelos de uso del tiempo en general, y en particular del modelo dominante en este campo, sugerido en Becker (1965). Pollak and Wachter (1975) deja algunas cuestiones abiertas sobre las cuales proponemos algunas respuestas en esta tesis doctoral; asimismo, este artículo generó algunas respuestas directas, como es el caso de Barnett (1977). Otra perspectiva interesante se encuentra en Gronau (1977), que usa un modelo teórico muy simple que ofrece interesantes implicaciones e interpretaciones de situaciones reales contrastadas empíricamente.

Pueden encontrarse algunas otras discusiones de interés en revisiones sobre la literatura, como en Flemming (1973) y en Juster and Stafford (1991). Un énfasis especial debe realizarse sobre Juster and Stafford (1991), que ofrece una atractiva revisión de todas las contribuciones realizadas hasta esa fecha tanto desde la perspectiva teórica como empírica. En lo que se refiere a la parte teórica que nos concierne, resulta muy interesante la parte en la que se ilustran los modelos intertemporales que incluyen el uso del tiempo en economía.

Se encuentran pocas contribuciones que empleen modelos dinámicos en el ámbito del uso del tiempo. Sin embargo, encontramos que Fischer (2001) analiza el fenómeno de posponer tareas a través del estudio de variables de uso del tiempo. Asimismo, podemos encontrar en Gonzalez-Chapela (2004) una tesis doctoral que recoge varios ensayos sobre la asignación del tiempo usando modelos dinámicos, entre los cuales podemos destacar el ensayo que finalizó siendo publicado en Gonzalez-Chapela (2007).

En el capítulo 2 se intenta formular un modelo ampliado sobre el uso del tiempo con respecto al de Becker (1965). Dicho modelo debe eludir el problema de la producción conjunta, apuntado en Pollak and Wachter (1975). Asimismo, intentamos ilustrar cómo este modelo ampliado funciona a través de un ejemplo en particular, mediante una ilustración relacionada con el efecto de las políticas dirigidas a incrementar la edad de jubilación en el tiempo total trabajado por un individuo durante su vida laboral.

### Sobre la sobrecarga en la elección

En las últimas dos décadas, aproximadamente, un número creciente de investigadores ha ido prestando atención a lo que se conoce como sobrecarga en la elección (*choice overload*) y la paradoja de la elección. En esencia, el problema de sobrecarga en la elección y la paradoja de la elección versan sobre lo siguiente: más opciones disponibles en el conjunto de elección crea más problemas que beneficios para el individuo que toma una decisión a medida que el conjunto de elección aumenta.

Podemos establecer que las publicaciones que han desencadenado el auge de esta línea de investigación son aquellas realizadas por Iyengar and Lepper (2000) y Schwartz (2000). Ambas, –de forma complementaria y casi simultánea, podríamos decir–, ofrecen tanto una enriquecedora disertación desde la perspectiva psicológica sobre la paradoja de la elección (Schwartz, 2000) como resultados empíricos en favor de la existencia de sobrecarga en la elección mediante estudios de campo en el mundo real y también en experimentos de laboratorio (Iyengar and Lepper, 2000).

Este asunto ha llamado la atencion tanto en psicología como en economía, si bien en economía de inicio ha formado parte de investigaciones relacionadas con el marketing y la economía de la empresa. A lo largo del crecimiento de este area de investigación, podemos encontrar varias publicaciones adicionales de los mismos autores mencionados al inicio, como Schwartz (2004), Schwartz (2005), Schwartz (2006) o Mogilner et al. (2008).

Más recientemente, la sobrecarga en la elección ha llamado la atención en economía experimental y economía del comportamiento, como se demuestra en Reutskaja (2008), Reutskaja and Hogarth (2009), Reutskaja et al. (2011) o Caplin et al. (2012). Asimismo, encontramos una variedad de análisis empíricos en contra (Scheibehenne et al., 2010) y a favor (Chabris et al., 2009) de la existencia de sobrecarga en la elección, interesantes discusiones teóricas con algunos argumentos empíricos como Dar-Nimrod et al. (2009), Markus and Schwartz (2010) o Grant and Schwartz (2011), y aplicaciones a temas de actualidad en economía (Iyengar and Kamenica, 2010; Kamenica et al., 2011).

En el capítulo 3 se pretende aplicar la teoría general formulada en el capítulo 2 a situaciones donde tiene lugar la sobrecarga en la elección descrita por los psicólogos sociales en Iyengar and Lepper (2000) y Schwartz (2000). Las variables de uso del tiempo han sido sugeridas como relevantes a la hora de estudiar el asunto de la sobrecarga en la elección. Por tanto, esperamos obtener resultados formales, bajo un enfoque de uso del tiempo, sobre algunos fenómenos relacionados con la sobrecarga de la elección (primordialmente, el efecto parálisis y la paradoja de la elección); estos resultados esperan ser conseguidos usando el análisis económico desde el problema de elección del consumidor enfocado desde una perspectiva de uso del tiempo.

Posteriormente, en el capítulo 4 perseguimos nuestro tercer objetivo: mostrar evidencia empírica para contrastar los resultados obtenidos en el capítulo 3. Para realizarlo, desarrollamos un análisis numérico del modelo en el capítulo 3, y también aplicamos dicho análisis a un estudio de caso con datos reales de precios para diferentes opciones disponibles en el mercado de un tipo de producto. Estos resultados numéricos son robustos, y muestran evidencia en favor de nuestros resultados teóricos relacionados con la sobrecarga en la elección.

# Resumen

La microeconomía teórica ha centrado su atención en mayor o menor medida en modelos que incluyen el uso del tiempo como una variable explicativa

de manera endógena. Así, encontramos modelos como el conocido modelo de consumo-ocio. Un modelo tan simple ha generado resultados valiosos, por ejemplo, en el campo de la economía laboral, entre otros. Los modelos económicos teóricos sobre el uso o la asignación del tiempo comenzaron con Gary S. Becker, quien posteriormente fue laureado con el Premio Nobel en Economía por ésta y otras muchas contribuciones. Becker atrajo la atención hacia este campo de investigación. En el capítulo 2 mostramos cómo el modelo propuesto por Becker supone una importante generalización del modelo de consumo-ocio. Sin embargo, estos avances también generan ciertos problemas, conocidos en la literatura económica como la necesidad de rendimientos constantes de escala en la tecnología del hogar y la ausencia de producción conjunta en esa misma tecnología, es decir, en la producción de bienes y servicios por el hogar (en el límite, por un miembro del mismo). En este mismo capítulo desarrollamos un modelo ampliado sobre el uso del tiempo que permite considerar el principal problema, que no es otro que el de la producción conjunta tanto en bienes como en usos del tiempo. Un estrucura tan grande como la desarrollada en este capítulo es ilustrada haciendo uso de una simplificación exagerada del modelo ampliado propuesto, cuyo propsito es analizar el efecto que tendría un aumento en la edad de jubilación en un contexto que abarca todo el perodo de vida laboral. Esta simple ilustración genera algunos efectos paradójicos que los actores políticos deberían tener en consideración a la hora de decidir la implementación de estas políticas dirigidas a incrementar la edad de jubilación.

En el capítulo 3 reducimos el modelo ampliado a un caso que de nuevo sólo considera variables de uso del tiempo con el propósito de analizar posibles situaciones de saturación en un individuo a la hora de elegir entre múltiples opciones de elección. Inicialmente nos referimos a un problema de elección que se centra en un producto para el cual existen diferentes opciones en el mercado (diferentes marcas, etc). El individuo debe elegir la manera en que distribuir su tiempo en tres posibles formas: hacer la compra, trabajar o tiempo libre. A mayor tiempo dedicado para hacer la compra, mejor precio encontrará en el mercado para dicho producto. Dada la restricción presupuestaria y las restricciones de tiempo para el individuo, este debe decidir la manera de distribuir su tiempo que maximice su bienestar. Un modelo simple como el propuesto genera diferentes soluciones, correspondientes a diversos perfiles de consumidores o individuos. El análisis de los diferentes perfiles se agrupa categorizando a los individuos o consumidores de la siguiente forma: consumidor ordinario, adictos al trabajo, amantes de las compras y consumidores no restringidos. Tras la discusión, demostramos y explicamos formalmente algunos fenómenos descritos desde el campo de la psicología social conocidos como saturación o sobrecarga en la elección, en concreto la paradoja de la elección y el efecto parálisis. Proponemos condiciones matemáticas que garantizan la presencia de estos fenómenos.

A lo largo del capítulo 4 mostramos evidencia empírica de los resultados teóricos sugeridos anteriormente haciendo uso de análisis numéricos. Partiendo de un modelo como el del capítulo 3, definimos la configuración específica del modelo que va a ser usada para generar dichos resultados empíricos. Proponemos un método para generar la estructura de precios en el mercado, así como la conducta de compra en términos de tiempo mínimo que debe ser empleado para hacer la compra por parte del consumidor. Asimismo, aplicamos un conocido algoritmo de optimización para calcular las soluciones óptimas en cada uno de los casos. El análisis lo desarrollamos de manera dual: por un lado consideramos una distribución estadística teórica para la distribición de precios en el mercado, mientras que por el otro hacemos uso de datos reales sobre precios de un producto. Para cada eventualidad, aplicamos el modelo en primer lugar sobre la casuística descrita en el capítulo 3, y en segundo lugar lo hacemos para diferentes perfiles de consumidor con los datos reales sobre precios, respectivamente. Para todos los casos en ambas eventualidades, observamos que los resultados están en línea con las predicciones teóricas del capítulo 3.

Varias conclusiones pueden ser extraídas a lo largo de esta tesis. En primer lugar, un modelo ampliado sobre el uso del tiempo que permita la producción conjunta es posible, y así lo proponemos. La presencia de producción conjunta en algunos casos –como el relativo al aumento de la edad de jubilación– sugiere implicaciones de interés que de otra manera no surgirían. Como resultado de aplicar el modelo ampliado a un problema microeconómico de uso del tiempo más reducido, demostramos y proponemos condiciones formales que garantizan la presencia de fenómenos relacionados con la saturación o sobrecarga en la elección, fundamentalmente la paradoja de la elección y el efecto parálisis que induce al consumidor a abandonar el problema de elección. En particular, concluimos que la propia estructura de precios en el mercado en conjunción con la conducta de compra de los consumidores pueden ser suficientes para contradecir un paradigma muy enraizado en las sociedades y economías modernas: el hecho de que más opciones sobre las que elegir es mejor para los individuos. Ilustramos cómo el bienestar de los individuos se verá reducido a partir de un cierto número de opciones de elección, lo que se conoce como la paradoja de la elección. Incluso en el caso de que la paradoja no tenga lugar, demostramos que como consumidores estamos abocados a sufrir saturación en la elección bajo condiciones muy probables; es decir, un consumidor racional no inspeccionará todas las opciones de elección que se le planteen. De la misma forma, demostramos por qué en algunas situaciones los consumidores eligen no elegir, es decir, el efecto parálisis. Estos efectos están contrastados por los resultados empíricos que se deducen de nuestro análisis numérico.

## Futuras líneas de investigación

El capítulo 2 nos ha sugerido la posibilidad de una versión menos simplista del modelo ampliado de uso del tiempo que hemos usado para ilustrar los efectos de una política que aumenta la edad de jubilación; la idea sería considerar más individuos (la sociedad en su conjunto) e introducir aspectos fiscales para estudiar diferentes efectos y redistribuciones que cada sistema de pensiones podría generar. Otro campo de investigación sería tratar de conseguir una versión dinámica del modelo ampliado sobre el uso del tiempo, lo cual supone un reto considerable. Por último, desde este capítulo, sería un reto interesante el tratar de producir una teoría de equilibrio general que incluyera el uso del tiempo; esto ayudaría para analizar de forma más rigurosa la eficiencia en el uso del tiempo en una economía.

El capítulo 3 nos deja varias posibles extensiones del modelo para investigaciones futuras. Destacamos aquí la prioridad que vamos a dar a la extensión del modelo que permite al consumidor elegir de manera endógena qué número de opciones, de entre el total dado, va a ser inspeccionado de verdad. Este modelo contribuiría en la línea de investigación mostrada en Iyengar and Kamenica (2010), Kamenica et al. (2011) y Caplin et al. (2012); tales contribuciones sugieren que incrementar el número de opciones, no solamente reduce la participación en el mercado y reduce el bienestar de los consumidores, sino que también afecta al tipo de opciones que son elegidas finalmente. Otro interesante efecto que debe ser estudiado en un futuro con más detalle debe ser el efecto categorización; deberíamos ser capaces de hallar condiciones formales para nuestro modelo que expliquen el efecto categorización, lo cual sería de destacable interés. También, considerar más categorías de diferentes de productos, –cada una con diferentes versiones del mismo producto, en la estructura del modelo–, sería de mucho interés para poder estudiar la presencia o no de efectos cruzados de sobrecarga en la elección —o por contra, sinergias– cuando se elige entre diferentes productos; esta última tarea puede que se fusione en algunos momentos con el estudio del efecto categorización ya mencionado. Por último, continuaremos con la línea ya iniciada que consiste en ofrecer una medida alternativa de bienestar basada en las distintas distribuciones del uso del tiempo realizadas por los individuos; debido a su complejidad y a lo preliminar del estado de esta investigación, esto no ha sido incluido en esta tesis doctoral.

El capítulo 4 y todo el análisis numérico llevado a cabo nos ha sugerido la posibilidad de realizar un trabajo similar cuando las extensiones del modelo teórico sugeridas en el capítulo 3 estén finalizadas. Además, cabe mencionar que durante el proceso de elaboración de este análisis numérico nos surgió la idea, mencionada anteriormente, que trata sobre la generación de una medida alternativa de bienestar basada en las distribuciones del uso del tiempo.

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